EQUATIONS WORKSHEET

1A

NAME ____

STARTING A SHAKE

PRINCIPLES

1. An *Equations* shake is played with 24 cubes, six of each color.
2. A cube of each color contains both digits and operation signs.

Color	Symbols on the Cube
red	0, 1, 2, 3, +, –
blue	0, 1, 2, 3, x, ÷
green	4, 5, 6, –, x, ^ (or * in older games)
black	7, 8, 9, +, ÷, √

3. There are no grouping symbols, such as (,), [,], {, or }, on the cubes. Each player may write these symbols in a Solution where needed.
4. Two- or three-person games will be played.
5. The Goal-setter for the first shake is determined like this:
   a. Each player rolls a red cube.
   b. The player rolling the highest digit sets the first Goal.
   c. A player rolling an operation cube is eliminated unless everyone rolls an operation.
   d. Players tied for high digit roll again until the tie is broken.
6. On the next shake, the Goal-setter is the person to the *left* of the previous Goal-setter.
7. The symbols that are rolled on the top faces of the cubes are called the *Resources* for the shake.

EXERCISES

*Circle the number of each **true** statement in #1-12.*

1. An *Equations* match may have only two players.
2. An *Equations* match may have four players.
3. For each shake, 24 cubes are used.
4. *Equations* cubes come in four different colors.
5. No symbol appears on more than one color of cubes.
6. The + sign appears on only the blue and red cubes.
7. The digit 9 appears only on the black cubes.
8. The Goal-setter for the first shake is chosen by a chance procedure.
9. On each shake after the first one, the Goal-setter is the player immediately to the right of the previous Goal-setter.
10. There are no symbols of grouping on the cubes.
11. Players may write any grouping symbols they need in their Solutions.
12. The symbols that are rolled face up form the Resources for the shake.
13. Which four digits have the *highest* chance of being rolled? ______________
14. Which two operation signs have the *least* chance of being rolled? _________

EQUATIONS WORKSHEET

1B

NAME ______________________________

SETTING THE GOAL

PRINCIPLES

The player who rolled the cubes must set a Goal by taking cube(s) from the Resources and moving them to the Goal section of the playing mat.

1. The Goal must equal a number.
2. It may use no more than **six** cubes.
3. It may contain only one- or two-digit numerals.
4. The + and – cubes mean addition and subtraction. They may *not* be positive or negative signs.
5. If a root sign (√) does not have an index in front of it, the index is understood to be 2.

EXAMPLES

	Legal Goal	Value		Legal Goal	Value
a.	13 + 25	38	b.	2 √ 49	7
c.	25 x 4	100	d.	75 – 48	27
e.	48 ÷ 12	4	f.	√ 16	4
g.	3 ^ 4	81	h.	23 + 8 – 5	26
i.	7 – 9	-2	j.	3 ÷ 4	3/4

	Illegal Goal	Why It Is Illegal
k.	+13	+ means addition, not positive.
l.	– 9	– means subtraction, not negative.
m.	125	Only one- or two-digit numerals may be used in the Goal.
n.	23 x	It does not equal a number.
o.	15 + 8 + 70	At most *six* cubes may be used in the Goal.

If the Goal-setter sets an illegal Goal, challenge **Impossible**. There is no way the Goal-setter can write a correct Solution for a Goal that has no legal value in *Equations*.

EXERCISES

Write the ***value*** *of each legal Goal. If a Goal is not legal, write an X in the answer blank.*

1.	54 ÷ 9	______	**2.**	150 x 2	______	**3.**	+27 – 6	______
4.	56 x 10	______	**5.**	√ 25	______	**6.**	–31	______
7.	10 ^ 3	______	**8.**	÷ 5	______	**9.**	2 √ 81	______
10.	98 - 25	______	**11.**	12x50x1	______	**12.**	7 + 8 + 13	______

13. The opponent sets an illegal Goal. What should you do? ______________________

MORE CHALLENGING EXERCISES

14.	7 ÷ 0	______	**15.**	23 √ 0	______	**16.**	0 ÷ 34	______

EQUATIONS WORKSHEET

1C

NAME ______________________________

OPERATION SIGNS

PRINCIPLES

1. + stands for addition and may not be used as a positive sign.
2. – stands for subtraction and may not be used as a negative sign.
3. x stands for multiplication.
4. ÷ means division.
5. * or ^ stands for raising to a power (exponentiation). [Note: Older games have * on the cubes. Newer games use ^.]
6. √ represents the root operation.

EXAMPLES

a. 8 + 7 = 15

b. +8 + 7 is illegal since + may not be used as a positive sign.

c. 8 – 7 = 1

d. 7 – 8 = -1

e. 8 + –7 is illegal since – may not be a negative sign.

f. 8 x 7 = 56

g. 9 ÷ 3 = 3

h. 3 * 2 or 3 ^ 2 means "3 to the second power" or "3 squared," which is 3 x 3 or 9.

i. 2 √ 9 means "the square root of 9," which is 3 because 3 x 3 = 9. The 2 is called the *index*. The √ sign must always have an index in front of it except when the index is 2. In this case, the index is optional. So √ 9 also means the square root of 9.

EXERCISES

Write the ***value*** *of each expression or, if it is not allowed in* Equations, *write* ***illegal***.

1.	5 + 7	________	**2.**	+8 x 6	________
3.	9 – 4	________	**4.**	– 6	________
5.	9 x 4	________	**6.**	8 ÷ 2	________
7.	8 x –2	________	**8.**	4 ^ 2	________
9.	7 ^ 1	________	**10.**	√ 4	________
11.	2 √ 1	________	**12.**	√ + 4	________
13.	√ 16	________	**14.**	1 ^ 4	________
15.	2 ^ 3	________	**16.**	4 √	________

MORE CHALLENGING EXERCISES

17. 3 √ 8 ______ **18.** 4 √ 81 ______ **19.** 5 √ 32 ______

EQUATIONS WORKSHEET

1D

NAME ________________________________

RULES FOR GOALS

PRINCIPLES

A Goal with two operation signs may have more than one value.

- The Goal-setter may fix the value by physically grouping the cubes of the Goal.
- If the Goal-setter does *not* group the cubes, *players may interpret the Goal in any legal way.*

There is no built-in order of operations for the Goal (or the Solution).

"Ambiguous" means *having more than one meaning.* A Goal is ambiguous if has two or more values.

EXAMPLES

Goal	Values	Goal	Values
2x3 +1	(2x3) + 1 = 6 + 1 = **7**	**2x 3+1**	2x (3+1) = 2 x 4 = **8**
2x3+1	(2x3) + 1 = **7** *or* 2x (3+1) = **8**	**4x5x6**	(4x5) x 6 = 4 x (5x6) = **120**
14÷0+2	14 ÷ (0+2) = 14 ÷ 2 = **7** (14 ÷ 0) + 2 is undefined.	**20–8–6**	(20 – 8) – 6 = 12 – 6 = **6** 20 – (8 – 6) = 20 – (2) = **20**

EXERCISES

Write all legal interpretations of each Goal. Give the value of each interpretation.

Sample 8–2÷2 **Interpretations** (8 – 2) ÷ 2 = 6 ÷ 2 = **3**; 8 – (2 ÷ 2) = 8 – 1 = **7**

1. 8+7–6 ________________
2. 7–5+2 ________________
3. 2x8–4 ________________
4. 10–4x2 ________________
5. 9+7x3 ________________
6. 18÷2–1 ________________
7. 9–3÷3 ________________
8. 3^2+1 ________________
9. 7+3^2 ________________
10. 11–5–3 ________________
11. 8÷4÷2 ________________
12. 5x4÷2 ________________

MORE CHALLENGING EXERCISES

13. √4+5 ________________
14. √5^4 ________________

EQUATIONS WORKSHEET

1E

NAME ________________________________

MORE RULES FOR SETTING THE GOAL

PRINCIPLES

Here are some more rules for the Goal.

1. When you set the Goal, say "Goal" when you are finished.
2. Once a cube touches the mat in the Goal section, you may not put it back into Resources. The cube must stay in the Goal. However, you may rearrange or re-group the cubes of the Goal as long as you have not said "Goal" yet.
3. You may not change a legal Goal once it has been set.
4. You have **two minutes** to set the Goal. If you do not finish setting the Goal within two minutes, you lose a point and have another minute to finish the Goal.
5. If you are not leading in the match, you may call "Bonus" and move a cube to Forbidden before setting the Goal. If the Goal-setter is in the lead and makes a Bonus move, the cube in Forbidden must be returned to Resources.

EXERCISES

Circle the number of each ***true*** *statement.*

1. The Goal-setter may play a cube to Required or Permitted before setting the Goal.
2. A legal Goal is not changed after it is set.
3. The Goal-setter indicates that the Goal is finished by saying "Goal."
4. You may rearrange the cubes on the Goal section of the mat as long as you have not said "Goal."
5. Once the Goal-setter puts a cube on the Goal section of the mat, that cube may not be put back in Resources.
6. The time limit for setting the Goal is one minute.
7. Before setting the Goal, the Goal-setter may call "Bonus" and play a cube to any of the three sections of the playing mat.
8. If the Goal-setter is leading in the match and makes a bonus move, the cube in Forbidden must be returned to Resources.
9. If you do not finish setting the Goal within the time limit, you lose a point and have another minute to finish the Goal.

EQUATIONS WORKSHEET

1F

NAME ____________________________________

RULES FOR MOVING

PRINCIPLES

1. After the Goal is set, players take turns making moves. To make a **move**, take a cube from the Resources and put it in Required, Permitted, or Forbidden.
2. **a.** If a cube is moved to **Required**, the symbol on that cube *must* be used in any Solution.
   b. If a cube is moved to **Permitted**, the symbol on that cube *may* be used in a Solution but does not have to be used.
   c. If a cube is moved to **Forbidden**, the symbol on that cube *must not* be used in any Solution. Note: If there are two x signs in Resources and one is played to Forbidden, the other x sign may still be used in any Solution.
3. Once a cube is legally played to one of the sections of the mat, it is not moved for the rest of the shake.
4. On your turn, if you are not leading in the match, you may make a **bonus move**. To make a bonus move, the Mover must
   a. say "Bonus,"
   b. move a cube from Resources to Forbidden,
   c. move a second cube to any of the three sections of the mat.
5. On your turn, you may not pass. However, instead of moving, you may challenge the latest Mover.

EXERCISES

*Circle the number of each **true** statement.*

1. Once a cube is legally played to Required, Permitted, or Forbidden, it is not moved for the rest of the shake.
2. Every cube played to Required must be used in any Solution.
3. Every cube played to Permitted may be used in a Solution.
4. A Solution may contain *no* cube in Permitted and still be correct.
5. No cube in Forbidden may be used in a correct Solution.
6. If a + is played to Forbidden, no + may be used in a Solution even if there are other + signs in Resources.
7. Any player may make a Bonus move on any turn.
8. A bonus move must be made to Forbidden.
9. You may not pass on your turn.
10. Any challenge is made against the last person to move.
11. If you do not first say "Bonus," you may not move two cubes on your turn.

EQUATIONS WORKSHEET

1G

NAME ____________________________________

RULES FOR SOLUTIONS

PRINCIPLES

To be correct, your Solution must obey these rules.

1. You write your Solution as an Equation in this form: *Solution* = *Goal*
2. The Solution (left side of your Equation) equals the interpretation of the Goal you write as the right side of the Equation.
3. The Solution must use the cubes correctly.
   a. It contains at least *two* cubes.
   b. It uses *all* the cubes in Required.
   c. It uses *no* cube in Forbidden.
   d. It may use one or more cubes in Permitted.
   e. It contains only one-digit numerals.
   f. The + and – cubes mean addition and subtraction, not positive or negative signs.
   g. If a √ has no number in front of it, the root is 2 (square root).
4. Since there is no fixed order of operations in *Equations*, you must write grouping symbols (such as parentheses) to show the order you want the operations done (both in the Solution and, if needed, the Goal).

EXAMPLES

1. Suppose the Goal is **7x5**. Then the following are **correct** Equations (assuming the cubes are used correctly).

   a. (6 ^ 2) – 1 = 7x5 **b.** 3 + (8 x 4) = 7x5 **c.** 7 x 5 = 7x5
   d. [(8 x 5) – 6] + 1 = 7x5 **e.** [(√9) x (6+5)] + 2 = 7x5 **f.** [7 x (8 + 2)] ÷ 2 = 7x5

2. For the Goal **8x4+3**, the following are **incorrect** Equations.

	Incorrect Solution	Reason
a.	–1 + (6 ^ 2) = (8x4)+3	Uses – as a negative sign, not subtraction.
b.	(10 x 5) + 6 = 8x(4+3)	Two-digit numeral in the Solution.
c.	8 x 5 – 5 = (8x4)+3	An opponent can group the Solution as 8 x (5 – 5).
d.	(6 x 6) – 1 = 8x4+3	An opponent can group the Goal as 8x(4+3).
e.	(9 x 4) + 2 – 3 = 35	Right side is the *value* of the Goal, not the Goal itself.

EXERCISES

*Suppose the Goal is **6x5+2**. Circle the number of each correct Equation below. Assume the cubes are used correctly.*

1. 8 x 4 + 0 = 32 **2.** (5 ^ 2) + 7 = (6x5)+2 **3.** (3 x 10) + 2 = (6x5)+2

4. +2 + (8x5) = 6x(5+2) **5.** -1+[3x(6+5)] = (6x5)+2 **6.** (7 x 6) + 3 – 3 = 6x5+2

7. 3 x 8 + 8 = (6x5)+2 **8.** [√(9+7)] x 8 = (6x5)+2 **9.** 6 ÷ (1 ÷ 7) = 6x(5+2)

*Suppose the Goal is **17-4-8**. Write the interpretation of the Goal (not its value) that makes each of these Solutions correct or write* None *if the Solution cannot equal the Goal.*

10. (2 x 2) + 1 = _______ **11.** (4 x 4) + 3 = _______ **12.** (5 ^ 2) – 4 + 0 = _______

EQUATIONS WORKSHEET

1H

NAME ________________________________

TIME LIMITS

PRINCIPLES

These are the time limits for *Equations*.

1.	Rolling the cubes	1 minute
2.	Making a variation selection This time limit does not begin until after the one minute for rolling the cubes.	15 seconds
3.	Setting the Goal	2 minutes
4.	First turn of the player to the left of the Goal-setter	2 minutes
5.	All other regular turns (including bonus moves)	1 minute
6.	Stating a valid challenge after picking up the challenge block	15 seconds
7.	Deciding whether to challenge Impossible with no cubes in Resources	1 minute
8.	Writing a Solution During this two minutes, the Third Party must decide whether to present a Solution.	2 minutes
9.	Checking a Solution	2 minutes

Use the one-minute sand timer to enforce time limits. If you are being timed, you have ten seconds *after the sand runs out* to complete what you must do. An opponent must count down the ten seconds at a reasonable pace ("1010, 1009, ..." and so on). If the ten seconds runs out before you finish, you lose one point and get another minute to complete the task. If not finished by the end of a ten-second countdown at the end of the added minute, you lose another point and forfeit what you are doing.

EXERCISES

Give the time limit for each action.

1. rolling the cubes ____________
2. setting the Goal ____________
3. making the first move after the Goal is set ____________
4. Third Party deciding whether to present a Solution ____________
5. writing a Solution ____________
6. stating a valid challenge after picking up the challenge block ____________
7. deciding to challenge Impossible with no cubes in Resources ____________
8. checking an opponent's Solution ____________
9. making a variation selection ____________

*Circle the number of each **true** statement.*

10. A one-minute sand timer is used to enforce time limits.
11. If you are being timed but do not finish what you must do when the sand runs out, you are immediately penalized a point.
12. If you lose a point for a time-limit violation, you get another minute to finish what you must do.
13. The most a player may be penalized for violating a time-limit is two points.

EQUATIONS WORKSHEET

1I

NAME ______________________________

CHALLENGES

PRINCIPLES

You may make either of two challenges against the person who just moved.

1. **Impossible**: This means the Challenger thinks that *no correct Solution* can be written even if any or all of the cubes in Resources are used.
2. **Now**: This means the Challenger thinks a Solution can be *written using the cubes on the mat* and, if needed, *one cube from Resources.*

You may *not* make a Now challenge in two situations.

1. With no cubes in Required and Permitted since a Solution must contain at least two cubes.
2. With fewer than two cubes left in Resources.

In both cases, the challenge is set aside, and the player who tried to challenge Now is penalized one point.

Only the latest Mover may be challenged. In a three-player match, either of the other two players may challenge. To challenge, you must pick up the "challenge block" and say either "Now" or "Impossible."

Challenge **Impossible** when you think that *no* correct Solution can be written using:

- **a.** all the cubes in Required,
- **b.** any or all of the cubes in Permitted;
- **c.** any or all of the cubes still in Resources.

Challenge **Now** when you think *you* can write a Solution using:

- **a.** all the cubes in Required;
- **b.** any or all of the cubes in Permitted;
- **c.** **one** cube from Resources, if needed.

EXERCISES

Circle the number of each ***true*** *statement.*

1. You may challenge only on your turn.
2. Only the latest Mover may be challenged.
3. To challenge, a player must touch the challenge block.
4. Any Solution written after a Now or Impossible challenge must use all the cubes in Required.
5. After an Impossible challenge, a Solution may use any cubes in Resources.
6. After a Now challenge, a Solution may use any cubes in Resources.
7. After a Now challenge, a correct Solution might be written without using *any* Resource cubes.
8. A Now challenge is illegal when no cubes are in Required and Permitted.
9. A Now challenge may be made with only one cube left in Resources.
10. A Now challenge may be made with no cubes left in Resources.
11. A player who makes an illegal challenge is penalized a point.

EQUATIONS WORKSHEET

1J

NAME ______________________________

WRITING EQUATIONS

PRINCIPLES

After a valid challenge, at least one player must write an Equation.

1. After a **Now** challenge,
   - the Challenger *must* present an Equation.
   - the Mover may *not* present an Equation.
   - the Third Party *may* present an Equation.
2. After an **Impossible** challenge,
   - the Challenger may *not* present an Equation.
   - the Mover *must* present an Equation.
   - the Third Party *may* present an Equation.

If you challenge **Impossible**, you are saying that *no* correct Equation can be written. You, the Challenger, are *daring* the Mover to write an Equation. So the Mover must do so to prove you wrong. Note: The Goal may be challenged **Impossible** as soon as it is set.

If you challenge **Now**, you are saying that *you* can write a correct Equation using the cubes on the mat plus one cube from Resources (if needed). So you, the Challenger, must write an Equation after challenging Now.

In a three-player match, the Third Party must decide whether he will present an Equation. If writing an Equation, the Third Party must satisfy the same requirements as the other player (Challenger or Mover) who must write an Equation.

a. After an Impossible challenge, the Third Party may use any cubes in Resources.
b. After a Now challenge, the Third Party may use at most *one* Resource cube.

An Equation-writer must circle the Equation to be checked. If this is not done, the writer must do so immediately when asked by an opponent.

EXERCISES

*Circle the number of each **true** statement in Exercises **1-9**.*

1. At least one player must write an Equation after every valid challenge.
2. After making a Now challenge, the Challenger must present an Equation.
3. It is possible after an Impossible challenge that both the Challenger and the Third Party present Equations.
4. An opponent may challenge Impossible right after the Goal-setter has set the Goal.
5. When you Challenge Impossible, you dare the Mover to write a correct Equation.
6. If writing an Equation, the Third Party must satisfy the same requirements as the other player (Challenger or Mover) who must write an Equation.
7. If presenting an Equation after a Now challenge, the Third Party may use as many cubes from Resources as she wishes.
8. An Equation that is not circled when presented to opponents is automatically incorrect.
9. A player who does not circle the Equation being presented has one minute to do so.

EQUATIONS WORKSHEET

1K

NAME ______________________________

LAST CUBE PROCEDURE

PRINCIPLES

When one cube is left in Resources, no player may challenge Now. Instead, this is what you do.

1. The player whose turn it is moves the last cube in Resources to Required or Permitted.
2. All players have two minutes to write a correct Equation.
3. During the first of these two minutes, an opponent may challenge Impossible against the player who moved the last cube. If this is done, the last Mover and the Third Party have the rest of the two minutes to finish an Equation.
4. If the Impossible challenge is not made, the situation is called a "forceout." Any player who writes a correct Equation scores **4**; anyone who does not present a correct Equation scores **2**.

If no challenge is made during a shake, eventually only two cubes will be left in Resources. If an Equation can be written using only *one* of those cubes (or neither of them), a player should challenge Now against the last Mover. (Or an opponent could challenge Impossible if he thinks no Solution is possible with the remaining two cubes.)

If no challenge is made, the next Mover should move one of the two cubes left in Resources to either Required or Permitted. This makes an Equation possible with one more cube. However, the Mover was *forced* to move the second-to-last cube to the mat. For this reason, **any Now challenge at this point is set aside**. However, you may make an Impossible challenge with one cube left in Resources if you think no one can write a correct Equation.

If no one challenges Impossible, follow the procedure listed in the Principles above. The shake should end in a tie with each player writing a correct Equation within two minutes.

Note: Moving the last cube in Resources to Forbidden is illegal procedure. Any challenge is set aside, and the cube is put back in Resources.

EXERCISES

*Circle the number of each **true** statement.*

1. An Impossible challenge may be made after any move regardless of how many cubes remain in Resources.
2. A Now challenge may be made after any move.
3. The last Resource cube is never played to the mat.
4. If two cubes remain in Resources but only one is needed in an Equation, an opponent should challenge Now against the last Mover.
5. If no cube remains in Resources and no Impossible challenge is made, any player who writes a correct Equation scores 6.
6. If no cube remains in Resources and no Impossible challenge is made, any player who does not present a correct Equation scores 2.
7. Moving the last Resource cube to Forbidden is illegal procedure.

EQUATIONS WORKSHEET

1L

NAME ________________________________

CHECKING EQUATIONS

PRINCIPLES

1. All Equations must be presented before any is checked.
2. Once you present an Equation to your opponent(s), you may make no corrections or additions even if the time for writing Equations has not ended.
3. Opponents have two minutes to check each Equation. In a three-player match, *both* opponents check a player's Equation during the *same* two minutes. No other Equation should be checked during this time.
4. Within the time for checking an Equation, opponents must either accept the Equation or show that it is incorrect.
5. Do not use the cubes in Required, Permitted, and Resources to form the Solution being checked. This avoids arguments over where cubes were played on the mat.

EXERCISES

Circle the number of each ***true*** *statement in* ***#1-6****.*

1. One Equation may be checked while another player is still working on his Equation.
2. Only one Equation should be checked during a single two-minute time limit.
3. An Equation is correct if no opponent shows that it is incorrect.
4. After an Impossible challenge, the Mover's Equation is always checked before the Third Party's.
5. You should not use the cubes in Required, Permitted, and Resources to form the Solution being checked.
6. When two players are checking an Equation, one player gets two minutes to check it, then the other player gets two minutes to check it.

Player ***X*** *challenges* Impossible *against* ***Y****. The Third Party,* ***Z****, sides with* ***Y****. Circle the number of each* correct way ***X****,* ***Y****, and* ***Z*** *can check Equations.*

7. **X** and **Y** check **Z**'s Equation, then **X** and **Z** check **Y**'s Equation.
8. **X** checks **Y**'s Equation while **Y** checks **Z**'s Equation.
9. **X** and **Z** check **Y**'s Equation, then **X** and **Y** check **Z**'s Equation.

A three-player shake ends in a forceout. *Players* ***A****,* ***B****, and* ***C*** *all present Equations. Circle the number of each* correct *way* ***A****,* ***B****, and* ***C*** *can* begin *checking Equations.*

10. **A** check **B**'s Equation while **B** checks **C** and **C** checks **A**.
11. **A** and **C** check **B**'s Equation while **B** times them.
12. **A** and **B** check **C**'s Equation while **C** checks **A**'s Equation.
13. **B** and **C** check **A**'s Equation while **A** times them.

EQUATIONS WORKSHEET

1M

NAME ______________________________

CHECKING THE CUBES

PRINCIPLES

1. After a **Now** challenge, a correct Solution must use *all* the cubes in Required, any of the cubes in Permitted the writer wishes, *no* cubes in Forbidden, and at most *one* cube in Resources.
2. After an **Impossible** challenge, a correct Solution must use *all* the cubes in Required, any of the cubes in Permitted the writer wishes, *no* cubes in Forbidden, and *any* cubes in Resources the writer wishes.

EXERCISES

Circle the number of each Equation that uses the cubes correctly.

1. **Challenge**: Now — **Equation**: (6 x 4) + 3 – 0 = 27

Resources: 2 3 5 9 + – √		
Forbidden	**Permitted**	**Required**
3 ÷	4	6 x 0 + 3

2. **Challenge**: Never — **Equation**: (5 ^ 2) – (6 + 3) x 1 = 8 x 2

Resources: 1 2 3 4 8 ^ + x √		
Forbidden	**Permitted**	**Required**
÷ +	7	5 – 3 x 6

3. **Challenge**: Never — **Equation**: [[(7 + 3)^2] + (8 x 4)] ÷ 2 = 66

Resources: + ÷ x 4 7 8		
Forbidden	**Permitted**	**Required**
2 4 – √		x 2 ^ 3 +

4. **Challenge**: Now — **Equation**: 9 x (2 – 1) x (3 + 3) = 54

Resources: + 0 3 3 4		
Forbidden	**Permitted**	**Required**
7 ÷ 8		x 9 – 1 x + 2

5. **Challenge**: Now — **Equation**: [(√4) + 7] x (8 – 3 – 0) = 50 – 5

Resources: 1 1 2 ÷ x +		
Forbidden	**Permitted**	**Required**
6 9 ÷	–	3 √ + 8 0 x 4 7 –

EQUATIONS WORKSHEET

1N

NAME ____________________________________

SCORING AFTER A CHALLENGE

PRINCIPLES

1. After a challenge, you are *correct* if:
   - **a.** you had to write or chose to write a correct Equation and did so, or
   - **b.** you did not have to write a correct Equation (someone else did) and no one wrote a correct Equation.
2. After a challenge, points are scored as follows.
   - **a.** If you are not correct, you score **2**.
   - **b.** A correct Challenger or Mover scores **6**.
   - **c.** A correct Third Party scores **6** if siding with the Mover or with an incorrect Now Challenger or scores **4** if siding with a correct Challenger.

EXERCISES

*Give the scoring (**6**, **4**, or **2**) for players **X**, **Y**, and **Z** in each situation.*

	X	*Y*	*Z*
1. *X* challenges Impossible against *Y*. *Z* decides not to present an Equation. *Y* does not present a correct Equation.	____	____	____
2. *X* challenges Impossible against *Y*. *Z* decides not to present an Equation. *Y* writes a correct Equation.	____	____	____
3. *X* challenges Impossible against *Y*. *Y* presents a correct Equation, but *Z*'s Equation is incorrect.	____	____	____
4. *X* challenges Impossible against *Y*. Both *Y* and *Z* write correct Equations.	____	____	____
5. *X* challenges Impossible against *Y*. *Z* writes a correct Equation, but *Y* does not.	____	____	____
6. *X* challenges Impossible against *Y*. Both *Y* and *Z* present incorrect Equations.	____	____	____
7. *X* challenges Now against *Y*. Both *X* and *Z* write correct Equations.	____	____	____
8. *X* challenges Now against *Y*. *X* writes a correct Equation, but *Z*'s Equation is incorrect.	____	____	____
9. *X* challenges Now against *Y*. *Z* writes a correct Equation, but *X* does not.	____	____	____
10. *X* challenges Now against *Y*. Both *X* and *Z* present incorrect Equations.	____	____	____
11. *X* challenges Now against *Y*. *Z* does not present an Equation. *X* writes a correct Equation.	____	____	____
12. *X* challenges Now against *Y*. *Z* does not present an Equation. *X* does not write a correct Equation.	____	____	____

EQUATIONS WORKSHEET

10

NAME ______________________________

FINDING A SOLUTION BEFORE SETTING THE GOAL

PRINCIPLE
The Goal-setter should never set a Goal without first having a Solution for that Goal.

EXAMPLE

Assume you are the Goal-setter and you rolled these Resources:

0 0 1 2 2 3 3 4 5 6 6 7 7 8 9 + + x x – ^ ÷ ÷ √

1. Form a two-digit number from the cubes; for example, **42**. *Don't set this Goal* until you have written a Solution for it.
2. To make a Goal that large, you probably need to multiply. So find two numbers that multiply to 42. **7 x 6** = 42 ← Write that Equation on your paper.
3. If you have time during the two-minute time limit for setting the Goal, try to find a longer Solution for 42. For example, multiply two numbers to give a two-digit number smaller than 42, then add the difference, like this: **(7 x 5) + 7** = 42.
   Notice there are a second 7 and a + in the Resources.┘
4. Another approach is to multiply two numbers to give an answer *larger* than 42, then subtract, like this: **(8 x 6) – 6** = 42 ← Are there an 8, two 6's, and a – in the Resources? Yes.
5. Now that you have several Solutions for **42**, set that number as the Goal.

EXERCISES

*Exercises **1-2** refer to the Resources listed above.*

1. Write a five-cube Solution for **42** that uses the ^ sign. ______________ = 42
2. Write a *second* five-cube Solution for **42** that uses the ^ sign. ______________ = 42

*Circle the number of each Solution in Exercises **3-6** below that is correct for the Goal of **42** with the Resources listed above.*

3. (4 x 9) + 6 **4.** (9 x 5) – 6 **5.** (√9) x 3 x 5 **6.** 7 x (5 + 1)

*For Exercises **7-16,** assume the Resources below have been rolled. Also, as Goal-setter, you have in mind this Goal: **81÷3***

Resources: **0 0 1 1 1 2 3 3 3 5 6 7 8 9 9 + + – – x x ^ ÷ ÷**

7. Write a three-cube Solution for the Goal that uses **x**. ____ **x** ____ = 81 ÷ 3
8. Write a three-cube Solution for the Goal that uses **^**. ____ **^** ____ = 81 ÷ 3
9. Complete this five-cube Solution for the Goal: **(7 x** ____**) +** ____ = 81 ÷ 3
10. Complete this five-cube Solution for the Goal: **(5 ^** ____**) +** ____ = 81 ÷ 3
11. Complete this five-cube Solution for the Goal: **(5 x** ____**) –** ____ = 81 ÷ 3
12. Complete this five-cube Solution for the Goal: **(6 ^** ____**) –** ____ = 81 ÷ 3

*Circle the number of each correct Solution for Goal **81 ÷ 3** and Resources above.*

13. 9 x 3 ÷ 1 **14.** (8 x 3) + 3 **15.** (9 – 6) x (4 + 5) **16.** (5 ^ 2) + (3 – 1)

EQUATIONS WORKSHEET

1P

NAME ________________________________

CHANGING YOUR SOLUTION

PRINCIPLES

- When the Goal is set, each player must write a Solution on her paper.
- Solutions must be revised based on the moves that are made.

EXAMPLE

Assume these Resources are rolled: **0 0 1 1 2 2 3 3 4 4 5 5 5 6 6 7 8 9 + x x – ^ √**

Player 1 (**P1**) sets the Goal as: **35** Cross out **3** and **5** in the Resources above.

Here are the Equations that each player comes up with:

P1: **7 x 5** = 35 **P2**: **(6 ^ 2) – 1** = 35 **P3**: **(8 x 4) + 3** = 35

The Exercises below continue this shake.

EXERCISES

1. **P2** plays **–** to Required. Which players, **P1** or **P3**, must revise Solutions ? ________
2. Suggest a way **P3** can use the – in his Solution without changing what he already has. **P3**'s revised Solution: **(8 x 4) + 3 –** ____ = 35
3. **P3** plays **x** to Required. **P1**, whose Solution is now **(7 x 5) – 0**, has no problem, but **P2** must revise his Solution. Complete **P2**'s revised Solution: **(6 x** ____**) – 1** = 35
4. **P1** plays **+** to Forbidden. This knocks out **P3**'s Solution since there are no more +'s in Resources. Complete **P3**'s revised Solution *using the same number in both blanks*:

   (8 x ___**) –** ___ = 35
5. **P2** plays **9** to Forbidden. Do **P1** or **P3** need to change their Solutions? _____.
6. **P3** plays **5** to Required. That forces ____ (which player?) to revise his Solution. He comes up with **(7 x 5) – 0**, which happens to be the same as ____'s Solution.
7. **P1** must be careful. If she plays either of two cubes to Required, **P2** can Challenge Now. Which two cubes should she *not* play to Required? ________
8. To wiggle out of the jam, **P1** plays √ to Forbidden. Does this force either opponent to change his Solution? ______
9. **P2** is now in the same position that **P1** was in. Not wanting to make a Solution possible with one more cube, **P2** plays a second **x** to Required. How did **P2** change his Solution to work in another **x**? **P2**'s revised Solution: **(7 x 5) –** __________ = 35
10. Stumped by the latest move, **P3** gives up and challenges Impossible. Which player *must* present a Solution? _____ Which player *may* present a Solution? _____

EQUATIONS WORKSHEET

2A

NAME ______________________________

ODD NUMBER OF CUBES IN SOLUTIONS

PRINCIPLE

The cubes in a correct Solution alternate between digits and operation signs, with two exceptions.

1. When √ is used without an index (root number) as in √9.
2. When certain variations (such as two-digit numerals or base *m*) are in effect.

These are typical Solutions.

(4 ^ 2) + (9 ÷ 3) = 19 [**7** cube Solution] (8 x 7) ÷ (9 – 8) – 0 = 56 [**9** cube Solution]

The basic rules allow only one-digit numbers in Solutions. Also, the + and – signs mean addition and subtraction, not positive and negative. This means that most Solutions start with a digit and alternate signs and digits, ending with a digit. Such a Solution contains an **odd** number of cubes, with one more digit than operation sign.

EXAMPLES

a. For the playing mat at the right, there is no way a Now challenge can be made at this point. Since three operation signs must be used, *four* digits are needed. But only two are played to the mat. *Two* more digits are needed.

PERMITTED	REQUIRED
	6 + 7 – x

b. For the playing mat at the right, an Impossible challenge should be made. Four signs in Required means *five* digits are needed in any Solution. However, only *four* digits are available in Required, Permitted, and Resources.

RESOURCES: 8 x –	
PERMITTED	**REQUIRED**
	6 + 7 x ^ x

EXERCISES

Circle each number where there is ***no way a Now challenge can correctly be made.***

1.

PERMITTED	REQUIRED
0	5 7

2.

PERMITTED	REQUIRED
–	x + 7

3.

PERMITTED	REQUIRED
6	1 8 9

4.

PERMITTED	REQUIRED
9 ^	6 1 4 x

Circle each number where ***an Impossible challenge should be made.***

5.

RESOURCES: 8 2 1 +	
PERMITTED	**REQUIRED**
6	0 ÷ 2 – 1 5 7

6.

RESOURCES: 4 3 x	
PERMITTED	**REQUIRED**
√ 0	+ x 1 4 7 9 6

7.

RESOURCES: 1 3 4 6	
PERMITTED	**REQUIRED**
–	+ 7 0 5

8.

RESOURCES: 2 1	
PERMITTED	**REQUIRED**
6	x ÷ 7 + –

EQUATIONS WORKSHEET

NAME ____________________

2B

ORDER OF OPERATIONS

PRINCIPLE

Certain math rules regarding the order of operations do not apply in the game of *Equations.*

1. Addition and subtraction are not automatically done left to right.
2. Multiplication and division are not automatically done left to right.
3. Multiplication and division are not automatically done before addition and subtraction.
4. Raising to a power (^ or *) is not automatically done before any other operation.

When you write your Equation, you must use grouping symbols (parentheses, brackets, or braces) to show the order of operations you want.

EXAMPLES

Expression	*Math Meaning*	Equations *Meanings*
a. 8 – 5 – 2	(8 – 5) – 2 = 3 – 2 = **1**	(8 – 5) – 2 = **1** *or* 8 – (5 – 2) = 8 – 3 = **5**
b. 7 x 5 + 2	(7 x 5) + 2 = 35 + 2 = **37**	(7 x 5) + 2 = **37** *or* 7 x (5 + 2) = 7 x 7 = **49**
c. 8 ÷ 4 x 2	(8 ÷ 4) x 2 = 2 x 2 = **4**	(8 ÷ 4) x 2 = **4** *or* 8 ÷ (4 x 2) = 8 ÷ 8 = **1**
d. 7 + 3 ^ 2	7 + (3 ^ 2) = 7 + 9 = **16**	7 + (3 ^ 2) = **16** *or* (7 + 3) ^ 2 = 10 ^ 2 = **100**
e. 9 + 6 ÷ 3 ÷ 2	9 + [(6 ÷ 3) ÷ 2] = 9 + (2 ÷ 2) = 9 + 1 = **10**	9 + [(6 ÷ 3) ÷ 2] = 9 + 1 = **10** *or* (9 + 6) ÷ (3 ÷ 2) = 15 x (2 ÷ 3) = **10** *or* [(9 + 6) ÷ 3] ÷ 2 = 5 ÷ 2 = **2.5** *or* 9 + [6÷(3÷2)] = 9 + [6x(2÷3)] = 9 + 4 = **13**

If you do not group your Equation correctly, an opponent may add parentheses or brackets to make your Equation wrong. More on that in Worksheet **2C**.

EXERCISES

Compute every possible Equations *value of each expression.*

1. 8 x 3 – 2 ____________ **2.** 7 – 4 – 3 ____________

3. 9 x 8 ÷ 4 ____________ **4.** 2 + 5 ^ 2 ____________

5. 2 – 1 + 8 ____________ **6.** 4 ÷ 2 ÷ 8 ____________

7. 5 + 6 ÷ 2 ____________ **8.** 9 ÷ 3 x 3 ____________

9. 9 – 7 + 3 x 2 ____________ **10.** 3 x 2 ^ 2 + 3 ____________

Circle the number of each expression that has only one *value.*

11. 7 x 6 x 2 **12.** 4 – 3 + 7 **13.** 5 + 1 + 8 **14.** 7 + 9 – 3

Add grouping symbols to either or both sides of each Equation to make it **correct**.

15. 7 + 9 x 6 = 61 **16.** 8 – 6 ÷ 2 = 4 – 3

17. 9 x 3 + 8 x 2 = 8 x 5 + 3 **18.** 7 + 1 x 8 ÷ 4 = 17 – 4 – 3

19. 7 + 4 – 0 ^ 2 = 8 + 3 ^ 2 **20.** 5 – 2 x 4 ^ 2 = 9 x 5 + 3

21. 2 ^ 3 – 1 x 4 = 3 + 5 x 5 **22.** 4 x 5 ^ 2 – 1 x 3 = 60

EQUATIONS WORKSHEET

2C

NAME ______________________________

AMBIGUOUS EQUATIONS

PRINCIPLES

A mathematical expression that may equal more than one value is **ambiguous**. Either side of an Equation, Solution or Goal, may be ambiguous. An opponent may rewrite the ambiguous Equation on his paper and put grouping symbols anywhere that makes sense in order to make the Equation incorrect.

EXAMPLES

Equation	Ambiguous?	Comment
a. 9 x 5 + 2 = 47	yes	An opponent may interpret this Solution as 9 x (5 + 2), which equals 63, not 47.
b. 6 x 5 ÷ 2 = 15	no	Both interpretations of the Solution, (6x5) ÷2 and 6x(5÷2), equal 15.
c. (5 x 3) + 2 = 4 ^ 2 + 1	yes	An opponent may group 4^2+1 as 4^(2+1) so that it does not equal (5 x 3) + 2.
d. (3 + 1) ^ 2 + 1 = 4 ^ 2 + 1	yes	An opponent may add grouping symbols to both sides to make it wrong, like this: [(3 + 1) ^ 2] + 1 = 4 ^ (2 + 1)
e. (7+5) ÷ 6 x 1 = 4 ÷ 0 + 2	yes	An opponent may group the Goal as (4÷0)+2, which has no value.

An opponent may not put grouping symbols in an Equation where they interfere with the writer's grouping.

Example Suppose the Equation is (6 x 5) + (3 x 2) = 36. An opponent may *not* interpret the Equation like this: (6 x **[**(5) + (3)**]** x 2) = 36. The added parentheses and brackets undo the original groupings.

EXERCISES

If possible, add grouping symbols to either or both sides of each Equation so that it is ***incorrect****. If there is no way to do so, write* **Correct**.

1. 7 x 3 + 2 – 0 = 23

2. (9 + 7) x 8 ÷ 8 = 4 ^ 2

3. (9 – 4) x (6 – 3) = 5 x 4 – 1

4. [(4 + 1) x 9 + 4] ÷ 1 = 65

5. (8 + 4) ÷ (2 x 2) ^ 1 = 9 – 6

6. 8 x 7 + 0 x 6 + 4 = (5 x 10) + 6

Grouping symbols have been added by an opponent in hopes of making each Equation incorrect. Circle the number of each Equation where the opponent's grouping is ***(a)*** *legal and* ***(b)*** *makes the Equation incorrect.*

	Original Equation	Equation with Opponent's Grouping
7.	(4 x 3) – 2 + 5 = 15	(4 x 3) – **(**2 + 5**)** = 15
8.	(9 + 7) ÷ (4 x 2) = 1 + 1	(9 + **[**(7) ÷ (4)**]** x 2) = 1 + 1
9.	8 ÷ 4 – [0 x (9 – 7)] = 10 – 7 – 1	8 ÷ 4 – [0 x (9 – 7)] = 10 – **(**7 – 1**)**
10.	(1 + 2) x (6 – 1) x (5 – 3) = 30	(1 + 2) x **[**(6 – 1) x (5 – 3)**]** = 30
11.	[(5 + 6) – 2 ^ (1 + 1) = 81	[(5 + 6) – **(**2 ^ (1 + 1)**)** **]** = 81

EQUATIONS WORKSHEET

2D

NAME ______________________________

ADDING AND SUBTRACTING ZERO

PRINCIPLES

Adding 0 to a number or subtracting 0 from a number does not change the value of the number. That is,

$a + 0 = 0 + a = a$ and $a - 0 = a$ where a is any number

Use these facts to create a "junk" expression that uses more cubes but does not change the value of a Solution.

EXAMPLES

For the Goal **12**, a simple Solution is **3 x 4**. This Solution can be expanded in many ways.

a. (3 x 4) + 0 = 12 **b.** (3 x 4) – 0 = 12 **c.** (3 x 4) + (6 – 6) = 12

d. (3 x 4) – (7 – 7) = 12 **e.** (3 + 0) x (4 – 0) = 12 **f.** (3 x 4) + (0 x 9) = 12
any number ┘

Note: In Equations **a** and **b** above, parentheses are not needed since either interpretation of 3 x 4 + 0 and 3 x 4 – 0 equals 12.

EXERCISES

*Circle the number of each Equation in Exercises **1-8** that equals the Goal* __48__. If a Solution can be grouped so that it does *not* equal 48, consider it as incorrect.

1. 8 x 6 + 0 = 48
2. (7 ^ 2) – 1 – 0 = 48
3. 8 x 6 + 9 – 9 = 48
4. (9 x 5 + 0) + 3 = 48
5. [(9 x 5) + 3] – 0 x 7 = 48
6. (6 – 0) x [8 + (0 x 5)] = 48
7. (8 x 6) + 7 – 5 – 2 = 48
8. 0 + (8 x 5) + 6 + 2 = 48

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
9.	20	+ x 0 4 5	______
10.	36	– ^ 0 2 6	______
11.	4	+ – ÷ 2 5 5 8	______
12.	13	– – + 0 0 6 7	______
13.	72	– – x + 2 4 6 8 9	______
14.	3	+ x √ 0 2 8 9	______
15.	35	– x ^ 0 5 7 9	______
17.	45	+ x √ 0 5 8 9	______
16.	81	+ x x ^ √ 0 2 5 6 8 9	______

EQUATIONS WORKSHEET

2E

NAME ______________________________

MULTIPLICATION BY ZERO

PRINCIPLES
0 times any number and 0 divided by any number (except 0) equals 0. That is,
a x 0 = 0 x *a* = *a* where *a* is any number, and 0 ÷ *a* = 0 where *a* is any number except 0.

This principle, together with the addition or subtraction of 0 (Worksheet **2D**), can be used to lengthen Solutions.

EXAMPLES

For the Goal **21**, a simple Solution of **7 x 3** can be expanded in many ways.

a. (7 x 3) + (8 x 0) = 21 (8: any number)
b. (7 x 3) – (0 ÷ 4) = 21 (4: any number except 0)
c. (7x3)–[(5–5)x(2÷9)] = 21 (2÷9: any number)

As usual, leaving out grouping symbols can make an Equation wrong. For instance, suppose Solution **a** above were written like this:

(7 x 3) + 8 x 0 = 21

An opponent could group it to make it wrong: **[**(7 x 3) + 8**]** x 0 = 0, not 21

EXERCISES

Circle the number of each Equation in Exercises ***1-6*** *that equals the Goal* **50**. If a Solution can be grouped so that it does *not* equal 50, consider it as incorrect.

1. (7 ^ 2) + 1 – (0 x 8) = 50
2. [(8 x 6) + 2] + (0 ÷ 9) = 50
3. (9 x 5) + [5 – (6 x 0 x 4)] = 50
4. (6 – 6) x 9 + (8 x 7) – 6 = 50
5. [(7 x 6) + 8] – (9 – 9) ÷ 5 = 50
6. (5 ^ 2) x 2 – (0 ÷ 6) = 50

MORE CHALLENGING EXERCISES

Use ***all*** *the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
7.	27	– x x 0 3 6 9	______________
8.	15	+ + ÷ 0 4 7 8	______________
9.	2	– + x ÷ 4 5 7 7 8	______________
10.	36	+ + x ÷ 0 3 5 7 8	______________
11.	1	+ x x ÷ ÷ 0 2 3 4 6 9	______________
12.	21	– x ÷ √ 0 7 8 9	______________
13.	4	– + ^ √ 0 2 7 8 9	______________
14.	88	+ – x √ 0 5 6 8 9	______________
15.	38	+ + x x ^ 0 2 2 3 5 8	______________

EQUATIONS WORKSHEET

2F

NAME ______________________________

MULTIPLYING AND DIVIDING BY 1

PRINCIPLES
Multiplying or dividing a number by 1 does not change the value of the number. That is,
$a \times 1 = 1 \times a = a$ and $a \div 1 = a$ where a is any number.

As is true with adding or subtracting zero, the above principle can be used to add "garbage" to Solutions.

EXAMPLES

For the Goal **12**, a simple Solution of **7 + 5** can be made longer.

a. 7 + 5 x 1 = 12
b. 7 + 5 ÷ 1 = 12
c. (7 x 1) + (5 ÷ 1) = 12
d. (7 + 5) ÷ (9 – 8) = 12
e. (7 + 5) x (8 ÷ 8) = 12
same number (not 0) in both places
f. (7 + 5) – (0 ÷ 1) = 12

In Equations **a** and **b** above, no parentheses are needed since either interpretation of each Solution equals 12. However, in Equation **c**, parentheses *are* needed since 7 x 1 + 5 ÷ 1 could be interpreted as 7 x (1 + 5) ÷ 1, which equals 42.

EXERCISES

*Circle the number of each Equation in Exercises **1-8** that equals the Goal **24**. If a Solution can be grouped so that it does not equal 24, consider it as incorrect.*

1. 8 x 3 ÷ 1 = 24
2. 6 x 1 x 4 = 24
3. (7 + 5) ÷ 1 x 2 = 24
4. (8 x 3) ÷ (7 – 6) = 24
5. [(5 x 4) + 4] x (3 ÷ 3) = 24
6. (6 + 0) x 4 ÷ 1 = 24
7. (9 x 2) + 6 x (1 ^ 2) = 24
8. 5 ^ 2 – 1 + 0 x 1 = 24

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
9.	17	÷ + 1 8 9	______
10.	16	^ x 1 2 4	______
11.	2	x ÷ – 5 5 7 9	______
12.	30	x ÷ ÷ 5 6 9 9	______
13.	4	– x ÷ 2 8 8 9	______
14.	43	+ x x 1 3 5 8	______
15.	42	x ÷ ^ 1 6 7 9	______
16.	√81	– + + x ÷ 2 3 4 4 5 9	______

EQUATIONS WORKSHEET

2G

NAME ______________________________

FIRST POWER AND FIRST ROOT

PRINCIPLES

The first power of any number is the number itself. The first root of any number is the number itself. That is,

a ^ 1 = *a* and 1 √ *a* = *a* where *a* is any number.

These rules can be used to "pad" Solutions.

EXAMPLES

For the Goal <u>32</u>, a simple Solution of **8 x 4** can be made longer.

a. 8 x 4 ^ 1 = 32
b. (8 ^ 1) x 4 ^ 1 = 32
c. 1 √ 8 x 4 = 32
d. (7 – 6) √ 8 x 4 = 32
e. 8 x 4 ^ (<u>3</u> ÷ <u>3</u>) = 32 (same number (not 0) in both places)
f. 1 √ 8 x 4 ^ (6 – 5) = 32

These Solutions contain only necessary parentheses. For example, Solution **d** equals 32 no matter where extra grouping symbols are placed, like this.

[(7 – 6) √ 8**]** x 4 = 32 or (7 – 6) √ **(**8 x 4**)** = 32

When in doubt, use extra grouping symbols to make sure opponents cannot group your Solution incorrectly.

EXERCISES

*Circle the number of each Equation in Exercises **1-6** that equals the Goal <u>44</u>. If a Solution can be grouped so that it does not equal 44, add symbols of grouping to make it incorrect.*

1. 4 x (6 + 5) ^ 1 = 44
2. 1 √ (7 x 6) + 2 = 44
3. (7 ÷ 7) √ (9 x 5) – 1 = 44
4. 8 x 5 + 4 ^ (1 – 0) = 44
5. 1 √ (8 – 7) √ (7 ^ 2 – 5) = 44
6. 1 √ [(6 ^ 2) + 8] ^ (5 ÷ 5) = 44

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
7.	68	+ x √ 1 5 7 9	______
8.	30	+ x ÷ ^ 1 2 3 8 9	______
9.	51	+ + x ÷ ^ 2 5 7 7 8 9	______
10.	28	+ – x x √ 1 2 2 3 5 6	______
11.	13	+ – ^ √ 1 1 3 7 9	______
12.	14	– x ^ √ √ 1 3 5 6 7 8	______
13.	59	– – ^ ^ 1 5 6 7 9	______
14.	35	– x ÷ ^ √ 2 2 5 7 8 9	______

EQUATIONS WORKSHEET

2H

NAME ______________________________

POWERS AND ROOTS OF ZERO

PRINCIPLES
0 to any positive power is 0. Any positive root of 0 is 0. That is,
0 ^ *a* = 0 and *a* √ 0 = 0 where *a* is any *positive* number.

These principles, together with the addition or subtraction of zero, can be used to "pad" Solutions.

EXAMPLES

For the Goal <u>14</u>, the Solution **(5 x 2) + 4** can be made longer.

a. (5 x 2) + 4 – (0 ^ <u>9</u>) = 14 (any positive number ┘)
b. (5 x 2) + 4 + (<u>8</u> √ 0) = 14 (any positive number ┘)
c. (5x2) + 4 – (1√0 ÷ <u>7</u>) = 14 (any positive number ┘)

Do not reverse an expression like 8√0. 0√8 is undefined and makes any Equation automatically wrong.

EXERCISES

*Circle the number of each Equation in Exercises **1-6** that equals the Goal <u>31</u>. If a Solution can be grouped so that it does not equal 31, add grouping symbols to make it incorrect.*

1. 1 + (6 x 5) – (0 ^ 3) = 31

2. (7 x 4) + 3 + 1 √0 = 31

3. (5 ^ 2) + 6 – (1 – 1) ^ 7 = 31

4. [(3 – 3) √2] + (7 x 5) – 4 = 31

5. (9 x 3) + 4 + (5 √2 √0) = 31

6. (8 x 3) + 7 – (0 ^ 5 ^ 1) = 31

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
7.	5	+ + ^ ÷ 0 2 3 6 7	______________
8.	24	– x x √ 0 2 3 4 9	______________
9.	4	+ + – ÷ √ 0 1 5 6 7 8	______________
10.	33	+ – x ^ ^ 0 2 2 3 3 9	______________
11.	39	+ – – x ^ √ √ 2 2 3 4 5 6 7	______________
12.	3	+ – – ^ ^ √ 0 1 2 2 4 5 6	______________
13.	15	+ – x ÷ ^ 3 4 4 5 7 9	______________
14.	9	+ – – x ^ 1 2 5 6 6 8	______________

EQUATIONS WORKSHEET

21

NAME ____________________________________

POWERS AND ROOTS OF ONE

PRINCIPLES

1 to any power is 1. Any root (except 0) of 1 is 1. That is,
1 ^ *a* = 1 where *a* is any number and *a* √ 1 = 1 where *a* is any number except 0.

These principles, together with the multiplication or division of one, can be used to "pad" Solutions.

EXAMPLES

For the Goal <u>22</u>, the Solution **(5 x 4) + 2** can be padded.

a. (5 x 4) + 2 x (1 ^ <u>9</u>) = 22 (any number)

b. (5 x 4) + 2 ÷ (<u>8</u> √ 1) = 22 (any number except 0)

c. (5 x 4) + 2 ÷ [<u>7</u> √ (9 – 8)] = 22 (any number but 0)

d. (5 x 4) + 2 x [(<u>3</u> ÷ <u>3</u>) ^ 8] = 22 (same number (not 0) in both spots)

Do not reverse an expression like 8√1 since 1√8 is 8 and not 1.

EXERCISES

*Circle the number of each Equation in Exercises **1-6** that equals the Goal <u>41</u>. If a Solution can be grouped so that it does not equal 31, add grouping symbols to make it incorrect.*

1. 1 + (8 x 5) x (1 ^ 3) = 41

2. (7 x 6) – (4 √ 1) = 41

3. (6 ^ 2) + 5 ÷ (4 – 3) ^ 7 = 41

4. [2 √ (3 ÷ 3)] x (7 x 7) – 8 = 41

5. (9 x 5) – 4 ÷ (5 √ 2 √ 1) = 41

6. (8 x 6) – 7 x (1 ^ 5 ^ 2) = 41

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
7.	29	+ x ^ 1 4 7 8	____________
8.	4	+ – x √ ÷ 1 2 3 7 7 9	____________
9.	28	+ – ^ ÷ √ 2 3 5 6 8 9	____________
10.	16	+ – x x ^ ÷ 2 2 3 5 7 7 9	____________
11.	38	+ x x ^ ^ 1 2 5 5 6 8	____________
12.	77	+ x ÷ √ √ 1 3 4 5 6 7	____________
13.	57	– – x x ^ √ 1 3 4 5 6 7 8	____________
14.	1	+ – x x ^ 2 3 5 6 7 8	____________
15.	6	+ + + ^ √ √ 1 2 3 5 7 8 9	____________

EQUATIONS WORKSHEET

2J

NAME ____

LEAVING "HOLES" IN SOLUTIONS

PRINCIPLES

Whenever possible, leave a place in a Solution where *any* number can be entered without changing the value of the Solution.

EXAMPLES

a. For a Goal of **<u>20</u>**, a Solution might be **(4 x 5) + (0 x □)**.

"hole" ↲

Since 0 times any number is 0, the hole can be filled by any digit or by a "garbage" expres - sion. The Solution above might eventually grow into this:

(4 x 5) + [0 x (8 ÷ 2 – 3)]

these cubes filled the "hole" ↲

b. Other examples:

Goal	Solution
10	(6 + 7 – 3) x (1 * □)
72	(9 x 8) ÷ (□ √ 1) any number except 0 ↲
64	(4 * 3) x (□ ÷ □) same number (not zero) in both places ↲

This strategy has two advantages.

1. Any symbol an opponent plays to Required can be placed in the "hole" in your Solution.
2. Suppose your Solution is (4 x 5) + (0 x □). If all but one of the cubes 4, x, 5, +, 0, and x have been played to Required or Permitted, you can challenge Now if *any* digit is played to Required or Permitted. Also the second x can be replaced by ÷.

EXERCISES

*For the Goal shown, use **all** the Resources listed to write a Solution. However, leave at least one "hole" in the Solution. If there is a restriction on filling the hole (such as* not zero *or* positive number*), indicate the restriction.*

	Goal	Resources	Solution
1.	81	+ x * 0 2 9	____________
2.	65	+ x ÷ * 1 7 8 9	____________
3.	18	x ÷ ÷ √ 1 1 2 9	____________
4.	18	– + + √ 0 5 6 7	____________
5.	90	+ x x * 0 3 5 6	____________
6.	14	+ – ÷ √ 1 3 8 9	____________
7.	38	+ – ÷ x 0 2 5 8	____________

EQUATIONS WORKSHEET

2K

NAME ______________________________

ORDER OF OPERATIONS WITH √

PRINCIPLE

The root sign (√) applies to just the numeral immediately behind it by default unless the Equation-writer indicates otherwise with symbols of grouping. This rule applies to the Goal as well as the Solution.

EXAMPLES

Expression	*Default Meaning*	*Expression*	*Default Meaning*
a. √25+9 (Goal)	(√25) + 9 = 5 + 9 = **14**	**b.** √4–5	(√4) – 5 = 2 – 5 = **-3**
c. √√16+9 (Goal)	[√(√16)]+9 = (√4)+9 = **11**	**d.** 4√1+9	(4√1)+9 = 1 + 9 = **10**
e. 3√8x7	(3√8) x 7 = 2 x 7 = **14**	**f.** 3+√9x5	3+[(√9)x5]=**18** or [3+(√9)]x5=**30**

Remember

- For your Equation, you must use parentheses if you do *not* want the default (automatic) order of operations.
- As a checker, you may *not* put parentheses in another player's Equation to violate the rule listed above.

EXAMPLES

g. The following Equation is correct mathematically. There is no way a checker can add parentheses to make it wrong.

1√9 + (2 x 2) – (0 ÷ 7) = √36 + 7

h. The next Equation is wrong because the writer did *not* add necessary parentheses.

What was written: (8 ÷ 4) x (5 ÷ 5) = 3√7+1

What *should* have been written: (8 ÷ 4) x (5 ÷ 5) = 3√(7+1)

EXERCISES

Give the default *value of each expression.*

1. √64–7 ______ **2.** √9+7 ______ **3.** √4x5 ______

*If possible, add grouping symbols to either or both sides of each Equation to make it **incorrect**. If there is no way to make it wrong, write **correct**.*

4. 5 √ 1 ^ 6 ÷ 3 = √ 81 ÷ 9

5. (8 x 3) + 7 – 6 = 5 ^ √ 4

6. √ 4 + 5 – 3 + 9 = (5 x 3) – 2

7. 3 ^ 4 x 2 ÷ 2 – 5 √ 0 = 81

Add the minimum *number of grouping symbols to either or both sides of each Equation to make it **correct**.*

8. √ 9 + 2 + 4 – 3 = √ 25 + 11

9. 1 + 2 √ 8 x 5 + 6 = √ 9 + 13

10. 3 x √ 9 – 5 + 8 = 16 – 2

11. 8 ÷ 2 x √ 4 x 7 – 6 = 56 ÷ 7

12. √ 3 ^ 4 + 2 – 0 ÷ 7 = 4 √ 16 + 9

13. √ 5 ^ 2 + 4 x 6 x 8 ÷ 8 = 63 ÷ 9

EQUATIONS WORKSHEET

3A

NAME ______________________________

SUBTRACTING LARGER FROM SMALLER

PRINCIPLE

A negative number can be obtained by subtracting a larger number from a smaller number.

EXAMPLES

1. To obtain a Goal of ⁻2 ("negative 2" in mathematics but not in Equations), set **0 − 2** or **1 − 3** or **12 − 14**, and so on.
2. For a Goal like **0 − 2**, the Solution must also involve a larger number subtracted from a smaller. Thus possible Solutions might be 7 − 9 or 6 − (2 x 4). As usual, these could be "padded" as follows.

   7 − 9 + (0 x □) 6 − (2 x 4) x (1 * □)
   7 − 9 ÷ (□√1) 6 − (2 x 4) − (0 * □)
   ↑ not zero ↑ positive number

EXERCISES

Write all possible values of each Goal in Exercises 1-9.

1. 0 − 5 ________	2. 2 − 9 ________	3. 4+1−6 ________
4. 8 − 12 ________	5. 16 − 22 ________	6. 25−65 ________
7. 1−3+4 ________	8. 2−7+5 ________	9. 3−6−2 ________

Write the value of each Solution in Exercises 10-14.

10. (4 + 3) − (7 x 2) ______
11. (4 ÷ 2) − (6 + 3) ______
12. (5 * 2) − (9 x 7) + 0 ______
13. [2 √(8 + 8)] − (4 * 2) x (5 ÷ 5) ______
14. [0 * (7 x 6)] − 8 ÷ (9 √1) ______

MORE CHALLENGING EXERCISES

Use <u>all</u> the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
15.	1 − 9	− − * √ 0 1 2 3 8	________________
16.	0−(9x4)	− x x ÷ * 2 5 5 6 8 9	________________
17.	2 − 5	+ + − x * 0 3 6 7 8 9	________________
18.	7 − 7	+ − x * 0 2 3 5 6	________________
19.	5 − 25	− x x √ 1 2 4 6 8	________________
20.	24 − 38	+ + − − * √ 1 5 5 6 7 8 9	________________

EQUATIONS WORKSHEET

3B

NAME ______________________________

ADDING WITHOUT +

PRINCIPLE
Addition can be done by subtracting a negative number.

a + b = a - (0 - b)

EXAMPLES

1. In a Solution or in the Goal, instead of 8 + 3, use 8 - (0 - 3), 8 - (1 - 4), 8 - (2 - 5), and so on.
2. In place of 6 + (8 x 7) in a Solution , use 6 - [0 - (8 x 7)].

STRATEGY
If two or more - signs are available, play all +'s to Forbidden.

EXERCISES

Write the value of each Solution in Exercises 1-8.

1. 7 - (0 - 3) ________
2. 5 - (2 - 4) ________
3. 6 - [0 - (5 x 2)] ________
4. 9 - [(6 ÷ 3) - 7] ________
5. (2 x 3) - [0 - (8 ÷ 2)] ________
6. (7 x 2) - [(8 ÷ 2) - (3 x 5)] ________
7. (1 - 5) - (0 - 3) ________
8. (2 - 7) - [0 - (4 x 3)] ________

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
9.	12	- - 0 4 8	________
10.	5 x 3	- - 1 7 9	________
11.	7 + 6	x - - 2 2 3 9	________
12.	1	- - - 0 3 5 9	________
13.	0 - 10	- - - 1 2 5 8	________
14.	0-(5x4)	- - x x 0 1 3 4 8	________
15.	11 - 43	+ - * √ 0 2 4 5 9	________
16.	0-(9*2)	- - x * 1 3 4 8 8	________
17.	1-(2√9)	+ - - x * 0 1 3 5 6 7	________
18.	1 - 39	+ - - x 3 4 6 8 9	________

EQUATIONS WORKSHEET

3C

NAME ______________________________

NEGATIVE TIMES POSITIVE

PRINCIPLES

1. A negative number times a positive number equals a negative product.
2. A negative number divided by a positive number (or a positive divided by a negative) produces a negative quotient.

These rules can be used to write Solutions for negative Goals.

EXAMPLES

a. 8 + (0 – 7) = –56

b. For a Goal of <u>**0 – 24**</u>, a correct Solution is (1 – 4) x 8.

c. (7 + 5) ÷ (1 – 3) equals –6.

d. For a Goal of <u>**1 – 4**</u>, a correct Solution is (9 x 2) ÷ (2 – 8).

e. –7 + 8 is illegal since – may not be a negative sign.

Negative numbers can be used much more easily when the Upside-down cube variation is chosen. (See Worksheet **7E**.)

EXERCISES

*Write the **value** of each Solution in Exercises **1-6**.*

1. 7 x (3 – 8) ________ **2.** (1 – 6) x (3 + 5) ________

3. (7 + 9) ÷ (4 – 8) ________ **4.** (2 – 8) ÷ (3 + 3) ________

5. (2 – 7) x (8 – 3) ________ **6.** (2 + 3) x (7 – 4) ÷ (1 – 2) ________

*Write all possible values of each Goal in Exercises **7-12**.*

7. 2 x 1 – 6 ________ **8.** 9 ÷ 3 – 4 ________

9. 4 – 5 ÷ 1 ________ **10.** 7 – 9 x 3 ________

11. 8 ÷ 1 – 9 ________ **12.** 5 x 0 – 9 ________

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal.*

	Goal	Resources	Solution
13.	0 – 48	+ – x 1 3 5 7	________
14.	1 – 19	– x ÷ 2 4 4 9	________
15.	3–(8*2)	+ – – x 3 4 5 7 9	________
16.	2 – 5	+ – ÷ ÷ 1 2 4 6 8	________
17.	0 – 70	+ – – x * 1 2 4 5 7 9	________
18.	2 – 60	+ – – * 3 4 5 7 8	________

EQUATIONS WORKSHEET

3E

NAME ______________________________

POWERS OF POSITIVE INTEGERS

PRINCIPLES
To raise a positive integer to a power which is a positive integer, multiply the integer times itself the appropriate number of times.

$$a * n = \underbrace{a \times a \times \ldots \times a}_{n \text{ factors}}$$

EXAMPLES

1. 2 * 4 = 2 x 2 x 2 x 2 = 16
2. 3 * 3 = 3 x 3 x 3 = 27
3. For a Goal of **64**, possible Solutions are 8 * 2, 4 * 3, and 2 * 6.
4. A Goal of **2 + 3 * 4** can be interpreted as 2 + (3 * 4), which is 2 + 81 = 83, or as (2 + 3) * 4, which is $5^4 = 625$.

EXERCISES

1. Complete this table of powers.

x^1	x^2	x^3	x^4	x^5	x^6
2					
3					
4					
5					
6					
7					
8					
9					
10					

Write all possible values of each Goal in Exercises 2-9.

2. 4 x 1 * 3 ________
3. 5 - 3 * 3 ________
4. 8 ÷ 2 * 3 ________
5. 4 + 2 * 4 ________
6. 2 x 3 * 5 ________
7. 4 * 4 ÷ 2 ________
8. 5 * 3 - 2 ________
9. 2 * 5 + 2 ________

Write the value of each Solution in Exercises 10-15.

10. (9 - 5) * (2 x 2) ________
11. (2 * 6) - (7 x 5) + 0 ________
12. (8 x 7) - (9 * 2) ÷ 1 ________
13. 2 * [5 - (0 x ($2\sqrt{4}$ + 5))] ________
14. (3 * 4) - (2 * 3) + (0 * 9) ________
15. 1 * (8 + 7 x 2 - 1 - 0) ________

EQUATIONS WORKSHEET

3H

NAME ____________________

POWER OF A POWER

PRINCIPLE

(a * m) * n = a * (m x n) or, in algebraic terms, $(a^m)^n = a^{mn}$

EXAMPLES

1. (2 * 3) * 2 = (2*3) x (2*3) = 8 x 8 = 64, which is 2 * 6.
2. [(1 - 4) * 2] * 3 = 9 * 3 = 9 x 9 x 9 = 729, which is 3 * 6 and (1 - 4) * 6.
3. A Goal of **3 * 2 * 3** has two interpretations: (3 * 2) * 3, which is 9 * 3 = 729, or 3 * (2 * 3), which is 3 * 8 = 6561.
4. Suppose the Goal is **3 * 56**. This is a very large number. However, a Solution could be made using the principle above: 3 * 56 = (3 * 8) * 7 or (9 * 4) * 7 [since 3 * 56 = 9 * 28]. In the unlikely event that three *'s are available (one for the Goal and two for the Solution), a good strategy would be to play all x signs to Forbidden to force opponents to use a power of a power.

EXERCISES

Write all values of each Goal in Exercises 1-4.

1. 0 * 9 * 97 ______
2. 2 * 3 * 4 ______
3. 3 * 1 * 2 ______
4. 3 * 2 * 2 ______

Write the value of each Solution in Exercises 5-8.

5. [(7 - 5) * 2] * 3 ______
6. [(5 - 7) * 2] * 3 ______
7. (8 x 7) x (1 * 6 * 9) ______
8. (0 - 1) * 3 * 5 ______

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
9.	2 * 63	- * * 3 5 7 9	______
10.	3 * 2 * 2	+ - x √ 0 5 8 9 9	______
11.	0 - 1 * 3	- * * 5 7 8 9	______
12.	9 * 24	+ - * * 1 2 3 7 8	______

EQUATIONS WORKSHEET

4A

NAME ______________________

EQUIVALENT FRACTIONS

PRINCIPLE

Fractions like 1 ÷ 2, 2 ÷ 4, 3 ÷ 6, and so on, are equivalent. For this set of fractions, 1 ÷ 2 is the one that is in simplest form or "reduced to lowest terms." However, in Equations there is no rule that requires fractional Goals or Solutions to be in simplest form.

EXAMPLES

1. For a Goal of **4 ÷ 6**, possible Solutions are 2 ÷ 3, 6 ÷ 9, 8 ÷ (7 + 5), and so on.
2. Suppose the Goal is **11 ÷ 3**. Then a possible Solution is [(5 x 4) + 2] ÷ 6. Another is [(5 x 6) + 3] ÷ 9.

If the Goal is a fraction, then the Solution must equal a fraction equivalent to the Goal.
Remember that you can create an equivalent fraction by multiplying or dividing the numerator and denominator of a fraction by the same number (not zero). This principle can be used to "pad" Solutions.

3. Suppose the Goal is **3 ÷ 4**. Then a possible Solution is 6 ÷ 8. This Solution could be expanded to (6 x 2) ÷ (8 x 2) or (6 x 5) ÷ [(8 x (4 + 1))], and so on.

multiply by same number

EXERCISES

Write the simplest form of each fraction in Exercises 1-6.

1. 2 ÷ 8 ________ 2. 6 ÷ 8 ________ 3. 10 ÷ 15 ________
4. 21 ÷ 14 ________ 5. 24 ÷ 30 ________ 6. 70 ÷ 50 ________

Write the value, in simplest form, of each Solution in Exercises 7-14.

7. 5 ÷ (7 + 3) x (1 * 8) ________
8. 6 x 2 ÷ (9 - 1) ________
9. (8 + 7) ÷ (5 x 4) + (3 √0) ________
10. [(8 x 3) + 2] ÷ (2 x 4) ________
11. (7 x 6) ÷ [(9 x 5) + 3] ________
12. [(7 * 2) + 1] ÷ [(9 + 3) x 5] ________
13. (9 x 6) ÷ [8 x (5 + 1)] ________
14. (8 x 3 ÷ 2) ÷ (5 x 4 ÷ 2) ________

EQUATIONS WORKSHEET

4B

NAME ______________________________

ADDING AND SUBTRACTING FRACTIONS

PRINCIPLE

A fractional Goal can often be made by adding or subtracting two fractions.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \text{and} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

You can add fractions quickly by a "cross multiplication" method.

EXAMPLE

$\frac{2}{3} + \frac{4}{5}$ can be added like this:

a. Form the denominator of the sum by multiplying the denominators: 3 x 5 = 15.

b. Compute the numerator of the sum by cross multiplying:

$\frac{2}{3} \times \frac{4}{5}$ (2x5) + (3x4) = 10 + 12 = 22.

↑ for subtraction, put a minus here

c. The sum is 22 ÷ 15.

The Goal-setter can create a fractional Goal as follows.

1. In Elementary and Middle Divisions, pick Sideways as your variation. This allows you to divide by multiplying by the reciprocal. (See Worksheet **7A**.)
2. From the Resources, form two fractions that can be added or subtracted.
3. Set the Goal equal to either the sum or the difference of the fractions depending on the cubes available.

EXERCISES

*Use the cross-multiplication method to add or subtract the fractions in Exercises **1-6**. Simplify answers if necessary.*

1. $\frac{2}{5} + \frac{1}{3} =$ ______ **2.** $\frac{3}{4} + \frac{5}{7} =$ ______ **3.** $\frac{1}{7} + \frac{1}{9} =$ ______

4. $\frac{5}{6} - \frac{1}{5} =$ ______ **5.** $\frac{5}{9} - \frac{2}{7} =$ ______ **6.** $\frac{2}{5} - \frac{6}{7} =$ ______

Fill in the blank to make each Equation correct.

7. (2 ÷ 7) + (1 ÷ 2) = ____ ÷ 14 **8.** (1 ÷ ___) + (5 ÷ 9) = 29 ÷ 36

9. (___ ÷ 5) + (2 ÷ 7) = 31 ÷ 35 **10.** (2 ÷ 3) + (___ ÷ 8) = 17 ÷ 12

11. (___ ÷ 8) – (1 ÷ 3) = 13 ÷ 24 **12.** (4 ÷ 9) – (___ ÷ 5) = 4 ÷ 90

13. (2 ÷ ___) + (8 ÷ 9) = 56 ÷ 36 **14.** (3 ÷ 4) + (___ ÷ 8) = 2 – (1 ÷ 8)

EQUATIONS WORKSHEET

4C

NAME ______________________________

MULTIPLYING FRACTIONS

PRINCIPLE
Multiply fractions by multiplying the numerators and multiplying the denominators. That is,

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

EXAMPLES

1. $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$

2. $\frac{4}{9} \times \frac{7}{8} = \frac{28}{72} = \frac{7}{18}$

3. Suppose the Goal is **7 ÷ 18**. Then, from Example 2, a possible Solution is (4 ÷ 9) x (7 ÷ 8) or, if only one ÷ sign is available, (4 x 7) ÷ (9 x 8). The player with this Solution should play cubes to Required and Forbidden to prevent opponents from making simple Solutions like 7 ÷ (6 x 3), (6 + 1) ÷ (9 x 2), and so on.

4. Remember that you can create an equivalent fraction by multiplying the numerator and denominator of a fraction by the same non-zero number. Thus, a Solution of 3 ÷ 4 can be expanded to this. (3 x ☐) ÷ (4 x ☐) — same number in both places

   Notice that this technique requires only one ÷ sign, whereas expanding the Solution as follows requires two.

   (3 ÷ 4) x (☐ ÷ ☐) — same number in both places

EXERCISES

Multiply the fractions in Exercises 1-6. Express each answer in simplest form (lowest terms).

1. $\frac{1}{4} \times \frac{3}{5} =$ ______
2. $\frac{2}{3} \times \frac{7}{9} =$ ______
3. $\frac{4}{5} \times \frac{3}{8} =$ ______
4. $\frac{9}{2} \times \frac{6}{7} =$ ______
5. $\frac{2}{5} \times \frac{7}{3} =$ ______
6. $\frac{7}{4} \times \frac{9}{5} =$ ______

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
7.	10 ÷ 21	x ÷ ÷ 4 5 6 7	______
8.	80 ÷ 56	x x ÷ 4 5 7 8	______
9.	45 ÷ 72	+ x x ÷ 0 5 8 9 9	______
10.	56 ÷ 88	+ x ÷ ÷ 3 3 5 6 7	______

EQUATIONS WORKSHEET

4D

NAME ______________________________

DIVIDING FRACTIONS

PRINCIPLE
To divide a fraction by a second fraction, multiply by the reciprocal of the second fraction. That is,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

EXAMPLES

1. $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$
2. $\frac{4}{9} \div \frac{1}{3} = \frac{4}{9} \times \frac{3}{1} = \frac{12}{9} = \frac{4}{3}$
3. $\frac{5}{7} \div 3 = \frac{5}{7} \div \frac{3}{1} = \frac{5}{7} \times \frac{1}{3} = \frac{5}{21}$
4. $8 \div \frac{2}{3} = \frac{8}{1} \times \frac{3}{2} = \frac{24}{2} = 12$

EXERCISES

Write the reciprocal of each number in Exercises 1-4.

1. $\frac{2}{5}$ ______ 2. $\frac{7}{3}$ ______ 3. $\frac{1}{6}$ ______ 4. 4 ______

Divide the fractions in Exercises 5-10. Express each answer in simplest form (lowest terms).

5. $\frac{4}{7} \div \frac{3}{5} =$ ______ 6. $\frac{9}{4} \div \frac{2}{7} =$ ______ 7. $\frac{2}{3} \div \frac{1}{5} =$ ______

8. $\frac{5}{9} \div 5 =$ ______ 9. $\frac{4}{5} \div 8 =$ ______ 10. $7 \div \frac{2}{3} =$ ______

Give the value of each interpretation of the Goals in Exercises 11-14. Express each answer in simplest form.

11. 8 ÷ 3 ÷ 4 ____________ 12. 6 x 2 ÷ 5 ____________

13. 9 ÷ 6 x 7 ____________ 14. 7 ÷ 1 ÷ 4 ____________

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
15.	12	x ÷ ÷ * 1 2 3 5 8	______________
16.	72 ÷ 15	+ ÷ ÷ 1 3 5 6	______________
17.	2	+ − ÷ ÷ ÷ 1 1 1 4 5 6	______________
18.	6 ÷ 3 ÷ 2	− − * * 0 2 2 6 9	______________

EQUATIONS WORKSHEET

4E

NAME ______________________________

MULTIPLYING WITHOUT X

PRINCIPLE
Multiplication can be done by dividing by the reciprocal.
That is, $a \times b = a \div (1 \div b)$

EXAMPLES

1. $8 \times 4 = 8 \div (1 \div 4) = 32$
2. In place of 7×2 in a Solution, use $7 \div (1 \div 2)$, $7 \div (2 \div 4)$, $7 \div (3 \div 6)$, and so on.

STRATEGY
If two or more ÷ are available, play all x's to Forbidden. (There are also ways to divide without ÷, as will be shown in later worksheets. Also see the More Challenging Exercises below.)

EXERCISES

Write the value of each Solution in Exercises 1–6.

1. $9 \div (1 \div 3)$ ______
2. $7 \div (2 \div 8)$ ______
3. $3 \div [1 \div (2 + 4)]$ ______
4. $5 \div [(7 - 6) \div (4 + 4)]$ ______
5. $(2 \div 3) \div (1 \div 5)$ ______
6. $(5 \times 4) \div (2 \div (7 - 1)]$ ______

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
7.	5 + 1	÷ ÷ 3 4 8	______
8.	36	+ ÷ ÷ 3 5 7 9	______
9.	20	+ x ÷ ÷ 2 3 5 6 9	______
10.	3 ÷ 4	÷ ÷ ÷ 1 2 2 3	______
11.	12	− ÷ * 0 1 3 4	______
12.	35	− ÷ √ 2 3 5 7	______
13.	48	− − ÷ * √ 2 2 4 5 7 8	______
14.	14	+ − ÷ * √ 2 3 6 7 8 9	______

EQUATIONS WORKSHEET

4F

NAME ______________________________

MIXED NUMBERS

PRINCIPLE
A mixed number like $8\frac{1}{2}$ can be converted to an improper fraction, like $\frac{17}{2}$, and vice-versa. That is,

$$a + (b \div c) = [(a \times c) + b] \div c$$

EXAMPLES

1. For a Goal of **15 ÷ 4**, a possible Solution is 3 + (3 ÷ 4).
2. For a Goal of **2 + (1 ÷ 5)**, one Solution is (7 + 4) ÷ (6 − 1).

Two special cases of the above principle are these.

a) $\frac{n-1}{n} = 1 - \frac{1}{n}$ b) $\frac{n+1}{n} = 1 + \frac{1}{n}$

EXAMPLES

3. 4 ÷ 5 = 1 − (1 ÷ 5)
4. 6 ÷ 5 = 1 + (1 ÷ 5)
5. For a Goal of **20 ÷ 22**, a possible Solution is 1 − [1 ÷ (7 + 4)].
6. For a Goal of **42 ÷ 39**, one Solution is 1 + [1 ÷ (9 + 4)].

This technique is useful because several 1's usually turn up on every roll. (It works especially well with the sideways variation, as will be shown in a later worksheet.)

A player using this approach must make moves to Required and Forbidden to force opponents to abandon other Solutions, such as (7 x 2) ÷ (9 + 4) for the Goal of **42 ÷ 39**.

EXERCISES

Write each mixed number in Exercises 1–4 as an improper fraction.

1. $3\frac{2}{3}$ ______ 2. $2\frac{1}{5}$ ______ 3. $7\frac{5}{6}$ ______ 4. $8\frac{2}{7}$ ______

Write each improper fraction in Exercises 5–8 as a mixed number.

5. $\frac{21}{4}$ ______ 6. $\frac{47}{8}$ ______ 7. $\frac{33}{10}$ ______ 8. $\frac{70}{9}$ ______

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
9.	27 ÷ 5	+ − ÷ 1 2 5 6	______
10.	3 + (5 ÷ 7)	+ + x ÷ 1 2 3 6 8	______
11.	34 ÷ 11	+ + ÷ * 1 2 3 7 9	______
12.	72 ÷ 81	− ÷ ÷ 1 5 5 9	______
13.	64 ÷ 60	+ − x ÷ 1 3 5 8 9	______

EQUATIONS WORKSHEET

4G

NAME ______________________________

POWERS OF FRACTIONS

> **PRINCIPLE**
> To raise a fraction to a power (a positive whole number), raise the numerator to that power and the denomination to that power.

EXAMPLES

a. $\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$ **b.** $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

c. Suppose the Goal is **27 ÷ 75**. In simplified form, this is 9 ÷ 25. So a possible Solution is (3 ÷ 5) ^ 2.

d. For the Goal **27 ÷ 64**, a possible Solution is (3 ÷ 4) ^ 3. Another Solution is [1 – (1 ÷ 4)] ^ 3. [This second Solution is particularly good with the Sideways variation: (1 – ⊣) ^ 3.]

e. A player with one of these Solutions should play x signs to Forbidden to prevent opponents from making Solutions like (9 x 3) ÷ (8 x 8) = 27 ÷ 64.

f. Another approach is to start with a fraction to a power like this: $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Then subtract $\frac{1}{32}$ from 1 to get $\frac{31}{32}$. Set that fraction as the Goal or $\frac{62}{64}$, an equivalent fraction. With Sideways, make the Solution 1 – (∾ ^ 5).

g. Instead of *subtracting* a fractional power from 1, you can *add* it to 1. For example **f** above, you would set the Goal as 33 ÷ 32 and make the Solution 1 + (∾ ^ 5).

EXERCISES

*Raise each fraction in Exercises **1-6** to the indicated power.*

1. $\left(\frac{1}{7}\right)^2$ _____ **2.** $\left(\frac{1}{3}\right)^4$ _____ **3.** $\left(\frac{3}{5}\right)^3$ _____

4. $\left(\frac{4}{3}\right)^3$ _____ **5.** $\left(\frac{1}{2}\right)^5$ _____ **6.** $\left(\frac{5}{4}\right)^2$ _____

*Write a **three**-cube Solution for each Goal. Assume Sideways is in effect.*

7. 1 ÷ 81 __________ **8.** 4 ÷ 32 __________ **9.** 1÷64 __________

*Write a **five**-cube Solution with no padding for each Goal. Assume Sideways is in effect.*

10. 30÷32 ____________________ **11.** 27÷64 ____________________

12. 84÷81 ____________________ **13.** 69÷24 ____________________

14. 65÷32 ____________________ **15.** 98÷50 ____________________

EQUATIONS WORKSHEET

4H

NAME ______________________________

SPECIAL FRACTIONAL GOALS

PRINCIPLE
A good strategy is to set a Goal that equals the sum or difference of powers of fractions.

EXAMPLES

1. Consider the expression $\left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^2 = \frac{1}{8} + \frac{9}{16} = \frac{2}{16} + \frac{9}{16} = \frac{11}{16}$

   The Goal-setter can do this arithmetic during the two minutes for setting the Goal and then set a Goal of **11 ÷ 16** or, even better since it somewhat disguises the strategy, **22÷32** or **33 ÷ 48**, and so on. To force a Solution like [(1 ÷ 2) * 3] + [(3 ÷ 4) * 2] the Goal-setter should play x signs and extra + signs to Forbidden.

2. Suppose the Goal is **31 ÷ 32** (or **62÷64**). A lengthy Solution is [(6 x 5) + 1] ÷ (8 x 4) or [(6 x 5) + 1] ÷ (2 * 5). However, a shorter Solution is 1 - [(1 ÷ 2) * 5]. The player with this Solution should play x signs and extra * signs to Forbidden. (This approach is particularly good with sideways cube.)

EXERCISES

Write the value of each expression in Exercises 1-6.

1. $\frac{3}{5} + \left(\frac{2}{3}\right)^2 =$ ______
2. $\left(\frac{1}{4}\right)^2 + \frac{5}{6} =$ ______
3. $\left(\frac{1}{2}\right)^5 + \left(\frac{3}{4}\right)^2 =$ ______
4. $\frac{7}{8} - \left(\frac{1}{2}\right)^2 =$ ______
5. $\left(\frac{2}{3}\right)^3 - \frac{2}{9} =$ ______
6. $\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^3 =$ ______

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
7.	47 ÷ 45	+ ÷ ÷ * 2 2 3 3 5	______
8.	63 ÷ 64	− − ÷ * 2 3 4 6 8	______
9.	75 ÷ 48	+ ÷ * 1 2 3 4	______
10.	9 ÷ 64	+ ÷ ÷ * * 1 2 2 3 4 8	______
11.	45 ÷ 72	− ÷ ÷ * 2 3 6 7 8	______
12.	35 ÷ 80	− − ÷ ÷ * * 1 2 2 3 3 6 8	______
13.	33 ÷ 56	− − ÷ * √ 2 3 5 7 8 9	______
14.	56 ÷ 81	− ÷ ÷ * 2 3 3 5 9	______
15.	29 ÷ 64	x − ÷ * √ 1 2 2 5 7 8	______

EQUATIONS WORKSHEET

5A

NAME ______________________________

SQUARE ROOTS

PRINCIPLE ↓2 understood

If $a^2 = x$, then a is the square root of x. In symbols: $a = \sqrt{x}$
In Equations, this is written $a = 2\sqrt{x}$ or just $\sqrt{x}$.

EXAMPLES

a. Since $2^2 = 4$, 2 is the square root of 4; that is, $2 = \sqrt{4}$.

b. Since $5^2 = 25$, 5 is the square root of 25; that is, $5 = \sqrt{25}$.

c. (-3)x(-3) = 9. So $(-3)^2 = 9$. -3 is *a* square root of 9. However, $\sqrt{9} = 3$, not -3. 3 is the *principal* (positive) square root of 9. 4 is the principal square root of 16.

d. $\sqrt{7}$ is not a whole number, and this expression is not allowed in Elementary Division in the Goal or Solution.

e. The Goal √9+7 has two values: (√9)+7 = 3 + 7 = 10; √(9 + 7) = √16 = 4.

EXERCISES

*Write the value of each expression in Exercises **1-6**.*

1. √36 ___ **2.** √64 ___ **3.** √49 ___ **4.** √81 ___ **5.** √100 ___ **6.** √144 ___

*Circle the number of each expression in Exercises **7-12** which does* not *equal a whole number.*

7. √16 **8.** √5 **9.** √3 **10.** √121 **11.** √60 **12.** √0

Write every possible value of each Goal.

13. √4+5 ________ **14.** 2√9–5 ________ **15.** 3–2√4 ________

16. 1+1√9 ________ **17.** √4x4 ________ **18.** √9÷9 ________

19. √4x9x4 ________ **20.** √16x25 ________ **21.** √9+√49 ________

Give the whole number value of each Goal.

Sample √98x50 = √(98x50) = √(49x2x50) = √(49x100) = (√49)x√100 = 7 x 10 = 70

22. √5x80 _____ **23.** √75x27 _____ **24.** √18x98 _____

25. √98x32 _____ **26.** √72x32 _____ **27.** √44x11 _____

28. √72x98 _____ **29.** √50x72 _____ **30.** √13x52 _____

*List all whole numbers you can put in the blank in each Goal to give a **whole number** value. For each number in the blank, list the whole number value that results.*

Sample √7+____ 9 → 4, 18 → 5, 29 → 6, 42 → 7, 57 → 8, 74 → 9, 93 → 10

31. √____–9 ______________________________

32. √24+____ ______________________________

33. √____÷3 ______________________________

34. √8x____^3 ______________________________

EQUATIONS WORKSHEET

5B

NAME ____

CUBE ROOTS

PRINCIPLE
If $a^3 = x$, then a is the <u>cube root</u> of x. In symbols: $a = {}^3\sqrt{x}$

EXAMPLES

1. Since $2^3 = 8$, 2 is the cube root of 8; that is, 2 = 3√8.
2. 3√64 = 4 since 4 x 4 x 4 = 64.
3. 3√0 = 0 since $0^3 = 0$.
4. 3√7 is not a whole number, and this expression is not allowed in Elementary Division in the Goal or Solution.

EXERCISES

Write the value(s) of each Goal in Exercises ***1-3****.*

1. 3√27 = ______ **2.** 3√7^3 = ______ **3.** 3√8^3 = ______

Circle the number of each expression in Exercises ***4-9*** *which does* not *equal a whole number.*

4. 3√9 **5.** 3√8 **6.** 3√27 **7.** 3√30 **8.** 3√81 **9.** 3√1

Write all **integer** *values of each Goal in Exercises* ***10-18****.*

10. 3√1+7 ________ **11.** 1+2√0 ________ **12.** 4-1√8 ________

13. 3√8x8 ________ **14.** 3√27x8 ________ **15.** 3√8+19 ________

16. 3√8√64 ________ **17.** 3√27√8 ________ **18.** 10^3√8 ________

List all integers > 1 you can place in the blank in each Goal to give a ***whole number*** *value. For each number in the blank, list the whole number value that results.*

Sample 3√7+____ 1 → 2, 20 → 3, 57 → 4

19. 3√____+9 ______________________________

20. 3√5+____ ______________________________

21. 3√4x____ ______________________________

22. 3√___÷3 ______________________________

23. 3√___x32 ______________________________

Mid/Jr/Sr: *Repeat the instructions for* ***#19-23*** *above. However, assume that the* ***Exponent*** *variation and* ***Sideways*** *are in effect, and that the sideways 3 in each Goal is the Exponent color as is the right-side up 3 in* ***#27****.*

24. ____+59ო ______________________________

25. ____÷ơ|ო ______________________________

26. ___ოx8ო ______________________________

27. 43x____ო ______________________________

EQUATIONS WORKSHEET

5C

NAME ____________________________________

HIGHER ROOTS

PRINCIPLE

If $a^n = x$ (for a positive integer n), then a is the <u>nth root</u> of x.
In symbols, $a = \sqrt[n]{x}$. In Equations this is written $a = n\sqrt{\;}x$.
"nth root of x"

EXAMPLES

1. Since $3^4 = 81$, 3 is the <u>fourth root</u> of 81; that is, $3 = 4\sqrt{\;}81$.
2. Since $2^5 = 32$, 2 is the <u>fifth root</u> of 32; that is, $2 = 5\sqrt{\;}32$.
3. $4\sqrt{\;}1 = 1$ since $1^4 = 1$. Similarly, $5\sqrt{\;}1 = 1$, $6\sqrt{\;}1 = 1$, and so on.
4. $4\sqrt{\;}0 = 0$ since $0^4 = 0$. Similarly, $5\sqrt{\;}0 = 0$, $6\sqrt{\;}0 = 0$, and so on.
5. $4\sqrt{\;}64$ is not a whole number and therefore is not allowed in Elementary Division in the Goal or Solution.

EXERCISES

NOTE: If necessary, refer to the table of powers you completed in worksheet 3E.

Fill in each box to correctly complete each equation in Exercises 1-6.

1. $4^4 = \square$ 2. $3^5 = \square$ 3. $1^9 = \square$

4. $\square^4 = 16$ 5. $\square^6 = 64$ 6. $\square^8 = 0$

Write the value of each expression in Exercises 7-16 if it is a whole number. If it is not, write *not a whole number*.

7. $4\sqrt{\;}16$ ________________ 8. $8\sqrt{\;}1$ ________________
9. $4\sqrt{\;}45$ ________________ 10. $9\sqrt{\;}0$ ________________
11. $5\sqrt{\;}32$ ________________ 12. $6\sqrt{\;}81$ ________________
13. $4\sqrt{\;}81$ ________________ 14. $6\sqrt{\;}64$ ________________
15. $5\sqrt{\;}90$ ________________ 16. $7\sqrt{\;}80$ ________________

List all possible one- or two-digit numbers that can fill the box in each Goal so that the Goal equals a whole number.

17. $4\sqrt{\;}\square$ ________________________ (Hint: 4 answers)
18. $5\sqrt{\;}\square$ ________________________
19. $6\sqrt{\;}\square$ ________________________
20. $7\sqrt{\;}\square$ ________________________
21. $8\sqrt{\;}\square$ ________________________
22. $9\sqrt{\;}\square$ ________________________

EQUATIONS WORKSHEET

5D

NAME ______________________________

ROOTS OF POWERS

PRINCIPLE
The "Umbrella Rule" can be used to evaluate a Goal that is a power of a root.

EXAMPLES

1. √4^6 = (√4)^6 = 2 ^ 6 = 64. But √4^6 can also be worked out like this:

   Divide the root (2) into the power (6). Then drop the root (2) and put the quotient (3) as the new power.

   √4^6 = 2√4^6 = 4 ^ **3** = 64.

   ← The arrow is shaped like an opened umbrella. Hence the "Umbrella Rule."
2. 3√2^6 = 2 ^ **2** = 4.

To work out a root of a power, do this:

Step	Example
1. Divide the root (3) into the power (12).	For 3√2^12, 12 ÷ 3 = 4.
2. Drop the root. (It becomes 1 and 1√2 = 2.)	3√2^12 = 1√2^4 = 2^4

WARNING FOR ELEMENTARY DIVISION

Parentheses must be used when writing the Goal in your Equation to make sure that each step in working out the Goal gives a whole number value.

Example

3√2^6 must be written as 3√(2^6) = √(2^2) = √4 = 2
Without the parentheses, an opponent may interpret the Goal as (3√2)^6. Since 3√2 is not a whole number, the Goal has no value in Elementary Division, and the Equation is incorrect.

EXERCISES

*Write the value(s) of each Goal in Exercises **1-8**.*

1. √9^4 = ______ **2.** 3√7^6 = ______ **3.** 3√2^9 = ______ **4.** 4√3^12 = ______

5. √10^6 = ______ **6.** 4√5^8 = ______ **7.** 5√8^10 = ______ **8.** 6√4^18 = ______

***Elementary** only: Circle the number of each Goal that can be grouped to make it undefined. In each case, put the parentheses that make the Goal undefined.*

9. √3^8 **10.** 3√27^6 **11.** 4√5^8 **12.** √4^10

List all whole numbers that can be put in the blank in each Goal to produce a whole number value. For each number in the blank, list the value it produces.

Sample 3√10^___ 0 → 1, 3 → 10, 6 → 100, 9 → 1000

13. 4√12^___ ______________________________

14. ___√4^10 ______________________________

15. 5√3^___ ______________________________

EQUATIONS WORKSHEET

6A

NAME ____________________________________

ZERO POWER

PRINCIPLE
x * 0 = 1 for any non-zero value of x. 0 * 0 is undefined.

Consider this sequence of powers.

decreasing exponents ↓

$2^5 = 32$ ←divide by 2
$2^4 = 16$ ←divide by 2
$2^3 = 8$ ←divide by 2
$2^2 = 4$ ←divide by 2
$2^1 = 2$ ←divide by 2
$2^0 = ?$

Based on the pattern of dividing by 2 to get the next smaller power, $2^0 = 1$.

EXAMPLES

1. 7 * 0 = 1
2. (0 − 3) * 0 = 1
3. (5 x 9) * 0 = 1
4. For a Goal of **1**, a Solution can be made of the form ☐ * 0. (any number except 0)
5. A Solution like 9 + 7 can be padded to 9 + 7 x [(☐) * 0].

EXERCISES

Circle the number of each Solution in Exercises 1-6 that equals a Goal of **40**. If a Solution can be grouped so that it does not equal 40, consider it incorrect and do not circle its number.

1. 8 x 5 x (9 * 0)
2. (6 * 2) + 4 ÷ 8 * 0
3. (9 x 5) − (3 + 2) x (2 + 1) * 0
4. (7 x 6) − 1 − (3 * 0)
5. (7 + 6) x 3 + (0 * 0)
6. (7 * 2) − 9 ÷ (3 x 1) * 0

MORE CHALLENGING EXERCISES

Use all the Resources listed to write a Solution for the given Goal.

	Goal	Resources	Solution
7.	(1−5)*0	+ x ÷ 3 4 5 7	____________
8.	5 ÷ 4	+ x ÷ * 0 2 3 7 8	____________
9.	4 √ 9 + 7	+ x * * 0 1 7 8 9	____________
10.	32	− x * * 2 3 3 5 9	____________
11.	3 ÷ 4	+ − ÷ ÷ * 2 5 6 7 8 9	____________
12.	1 * 99	+ − x * * 0 2 2 3 5 6	____________

EQUATIONS WORKSHEET

6N

NAME ______________________________

POWERS OF ROOTS

PRINCIPLE

$(\sqrt[n]{x})^n = \sqrt[n]{x^n} = x$ in all cases where $\sqrt[n]{x}$ is defined.

EXAMPLES

1. $(\sqrt{2})^2 = \sqrt{2}\cdot\sqrt{2} = \sqrt{4} = 2$

2. $(\sqrt[3]{4})^3 = \sqrt[3]{4}\cdot\sqrt[3]{4}\cdot\sqrt[3]{4} = \sqrt[3]{64} = 4$

3. $(\sqrt{2})^6 = \underbrace{(\sqrt{2})(\sqrt{2})}_{2}\underbrace{(\sqrt{2})(\sqrt{2})}_{2}\underbrace{(\sqrt{2})(\sqrt{2})}_{2} = 2\cdot 2\cdot 2 = 8$ ⟵ Notice $8 = 2^3$ so $2\sqrt{2} * 6 = 2 * 3$ (divide)

These last examples are for MJS only.

4. $(\sqrt[5]{-8})^5 = -8$ ⟵ Good with upside-down cube.

5. $\left(\sqrt[4]{\frac{1}{2}}\right)^4 = \frac{1}{2}$ ⟵ Good with side-ways cube.

6. $(\sqrt{3})^3 = \underbrace{\sqrt{3}\cdot\sqrt{3}}_{(\sqrt{3})^2}\cdot\sqrt{3} = 3\cdot\sqrt{3}$

7. $(\sqrt[3]{5})^5 = (\sqrt[3]{5})^3(\sqrt[3]{5})^2 = 5\cdot(\sqrt[3]{5})^2 = 5\cdot\sqrt[3]{25}$

EXERCISES

Write in simplest form the value of each expression.

1. $(\sqrt{7})^2$ ______ 2. $(\sqrt[3]{12})^3$ ______ 3. $(\sqrt[3]{8})^6$ ______ 4. $(\sqrt{3})^8$ ______

#5-12 are for MJS only.

5. $(\sqrt[3]{-9})^3$ ______ 6. $\left(\sqrt[7]{\frac{1}{3}}\right)^7$ ______ 7. $(\sqrt{5})^3$ ______ 8. $(\sqrt{3})^5$ ______

9. $(\sqrt[4]{7})^5$ ______ 10. $(\sqrt[3]{4})^5$ ______ 11. $\left(\sqrt[4]{\frac{1}{2}}\right)^6$ ______ 12. $(\sqrt[5]{-8})^7$ ______

Write in simplest form the value of each Goal.

13. √5 * 2 ______ 14. √6 * 4 ______ 15. 6 √ 2 * 18 ______

16. 4√10 * 8 ______ 17. 3 √7 * 6 ______ 18. 5 √ 3 * 15 ______

#19-23 are for MJS only.

19. √6 * 3 ______ 20. 3 √5 * 4 ______ 21. 4 √3 * 6 ______

22. Circle the letter of each correct Solution for a Goal of **2√5 * 3**.

a. (4+1)*(6÷4) b. (8−3)x(2√5) c. 5*(1÷2)x(4+1)

d. (3 ÷ 2) √5 e. (6÷9)√(8−3) f. (1÷3)√(5x5)

EQUATIONS WORKSHEET

6Q

NAME ______________________________

GOALS INVOLVING ROOTS

PRINCIPLE
A Goal that produces an integer root can be disguised to confuse opponents.

RECALL $\sqrt{(9 \times 4)} = (\sqrt{9}) \times (\sqrt{4}) = 3 \times 2 = 6$

EXAMPLE 1

The Goal **$\underline{\sqrt{16x81}}$** has two integer values.

$\sqrt{(16x81)} = 4 \times 9 =$ **36** and $(\sqrt{16})x81 = 4 \times 81 =$ **324**

But suppose the Goal is set as **$\underline{\sqrt{48x27}}$**.

Since $16 \times 3 = 48$ and $81 \div 3 = 27$, $48x27 = 16x81$. So $\sqrt{(48x27)} = \sqrt{(16x81)} =$ **36**.
Note that the second grouping of the Goal, $(\sqrt{48})x27$, does not produce an integer.

EXAMPLE 2

An opponent sets this Goal: **$\underline{\sqrt{18x50}}$**. Does it have any integer values?

a. Take the larger number, 50, and ask: "What number can I divide into 50 to give a perfect square?" Answer: 2 since $50 \div 2 = 25$.
b. But if I *divide* 50 by 2, I must *multiply* 18 by 2 to keep the overall value of the Goal the same.
c. So the Goal is the same as $\sqrt{(36x25)}$, which equals $(\sqrt{36})x(\sqrt{25}) = 6 \times 5$ or **30**.
CAUTION: You *cannot* group as $(\sqrt{36})x25 = 6 \times 25 = 150$ since the original Goal was $\sqrt{18x50}$ and $(\sqrt{18})x50$ does not give an integer value.

EXERCISES

*Write all possible **integer** values of each Goal.*

1. $\sqrt{27x12}$ ______ **2.** $\sqrt{18x50}$ ______ **3.** $\sqrt{48x75}$ ______

4. $3\sqrt{24x9}$ ______ **5.** $4\sqrt{64 \div 4}$ ______ **6.** $5\sqrt{96 \div 3}$ ______

*List all numbers that can fill the blank in each Goal to give an **integer** value. For each number you put in the blank, list the integer values that result. The first one is given for you: **2** in the blank produces a value of **12**.*

7. $\sqrt{72x}$____ $2 \rightarrow 12,$ ______________________________

8. $\sqrt{98x}$____ ______________________________

9. $\sqrt{31+}$____ ______________________________

10. $3\sqrt{4x}$ ____ ______________________________

11. $4\sqrt{\ }$ ____x3 ______________________________

12. $4\sqrt{\ }$ ____+9 ______________________________

EQUATIONS WORKSHEET

7A

NAME ______________________________

SIDEWAYS - I

PRINCIPLE (All divisions)

The Sideways variation says: *A cube representing a non-zero number may be used sideways in the Goal or Solution to equal the reciprocal of that number.*

NOTE: This variation is always in effect in Junior and Senior Divisions.

- The **reciprocal** of a number is 1 divided by that number. So the reciprocal of 2 is ½, the reciprocal of 3 is ⅓, the reciprocal of -4 is -1/4, and so on.
- Remember: 1 ÷ 0 has no value. So 0 has no reciprocal.
- The digit may be turned sideways either to the left or the right in the Goal. For example, 2_{sw} or 2_{sw} both mean ½.
- You may not use a sideways cube as part of a two-digit number. So expressions like $1\,5_{sw}$, $4_{sw}5$, and $4_{sw}5_{sw}$ are not allowed.
- When you write a sideways cube in an Equation, do it like this: 2_{sw}

EXAMPLES

a. 9 x 3_{sw} means 9 x (1/3) = 3. **b.** 8 ÷ 2_{sw} = 8 ÷ (1/2) = 8 x 2 = 16

c. 1 + 3 – 4_{sw} = 1 + 3 – (1/4) = 3 3/4 or 15/4.

d. 6_{sw} x 3_{sw} = (1/6) x (1/3) = 1/18 **e.** 8_{sw} ÷ 2_{sw} = (1/8) ÷ (1/2) = (1/8) x 2 = 1/4

EXERCISES

1. What is the only number that has *no* reciprocal? ______

*Write the **reciprocal** of each number.*

2. 8 ______ **3.** -5 ______ **4.** 1 ÷ 2 ______

*Write the **value** of each expression.*

5. 8 x 2_{sw} ______ **6.** 5_{sw} x 25 ______ **7.** 18 x 3_{sw} ______

8. 9 ÷ 3_{sw} ______ **9.** 10 ÷ 4_{sw} ______ **10.** 12 ÷ 2_{sw} ______

Give each value as an improper fraction like 15/4.

11. 5 + 3_{sw} _____ **12.** 6 – 2_{sw} _____ **13.** 7 – 6_{sw} _____ **14.** 4_{sw} + 9 _____

*Give **two** values for each Goal.*

15. 6+8x4_{sw} ______ **16.** 5x3–2_{sw} ______ **17.** 6÷2_{sw}÷3 ______

*With Sideways, write a Solution for each Goal using the cubes listed in Required and **one** more cube. Assume the cube you need is available in Resources.*

	Goal	Required	Solution		Goal	Required	Solution
18.	16	8, ÷	________	**19.**	4	x, 8	________
20.	1 ÷ 27	3, 9	________	**21.**	6 ÷ 10	5, x	________
22.	28 ÷ 3	3, +	________	**23.**	33 ÷ 6	–, 2	________
24.	41 ÷ 2	+, x, 2, 5	________	**25.**	11 – 3_{sw}	x, x, 4, 8	________

EQUATIONS WORKSHEET

7B

NAME ______________________________

SIDEWAYS - II

PRINCIPLE

The Sideways variation lends itself to useful strategies involving adding reciprocals.

Consider this problem: $\frac{1}{3}+\frac{1}{8} = \frac{8}{24}+\frac{3}{24} = \frac{11}{24}$ ← Sum of 3 and 8; ← Product of 3 and 8

STRATEGY BASED ON THIS PATTERN

1. Set 11 ÷ 24 as the Goal. Plan to make 3 + 8 your Solution.
2. Move any other ÷ signs to Forbidden as soon as possible. With no ÷ signs, an opponent who tries to "reproduce the Goal" with a Solution like (7+4) ÷ (8x3) is cooked.
3. Play a + sign to Required early in the shake. If an opponent plays 3 or 8 to Required or Permitted, challenge Now.
4. Also move x and ^ signs to Forbidden since you don't need any in your Solution.

OFFSHOOT OF THE STRATEGY

Disguise the Goal. Instead of 11÷24, set 22÷48 or 33÷72, which are equivalent fractions. This gives you options if, say, you don't have two 1's in Resources.

EXERCISES

Add the given fractions.

1.	2 + 3	______	**2.**	5 + 8	______	**3.**	4 + 7	______
4.	2 + 5	______	**5.**	6 + 7	______	**6.**	5 + 7	______

*Use the strategy above to write a **three-cube** Solution for each Goal.*

7.	9 ÷ 20	________	**8.**	15 ÷ 56	________
9.	28 ÷ 90	________	**10.**	33 ÷ 90	________
11.	44 ÷ 72	________	**12.**	20 ÷ 42	________

These questions refer to the strategy explained above.

13. Which operation signs should you play to Forbidden as soon as possible? _____
14. What sign should you play to Required early in the shake? ____
15. You should also move what other signs to Forbidden? _____

Use the strategy above to set a Goal and write a Solution from the Resources.

	Resources	Goal	Solution
16.	0 0 0 1 1 2 3 4 5 5 8 9 9 + + + − − ÷ ÷ x x ^ √	____ ÷ ____	____ (sw) + ____ (sw)
17.	0 0 1 1 2 2 3 4 6 7 7 8 9 + + − − − ÷ x x ^ ^ √	____ ÷ ____	____ (sw) + ____ (sw)
18.	0 0 1 1 1 2 2 3 3 5 6 7 9 9 + + − − ÷ ÷ x x x ^	____ ÷ ____	____ (sw) + ____ (sw)

EQUATIONS WORKSHEET

7C

NAME ____________________________________

SIDEWAYS - III

PRINCIPLE
Sideways lets you make a Solution for a fractional Goal without using a ÷ sign.

STRATEGY #1

1 – ∞ = 1 – (1/8) = 7/8 = 14/16 = 21/24 = 28/32, and so on.

So use this strategy:

1. Set the Goal as, say, 56 ÷ 64, which reduces to 7/8.
2. Move any other ÷ signs to Forbidden as soon as possible. This forces opponents to figure out the Solution 1 – ∞. If they can't, they may challenge Never.
3. Play a – sign to Required early in the shake. This sets up a Now challenge if an opponent plays 1 or 8 to Required or Permitted.

OFFSHOOTS OF STRATEGY #1

4. 1 + ∞ = 1 + (1/8) = (8/8) + (1/8) = 9/8 = 18/16 = 27/24, and so on.
5. 2 – ∞ = 2 – (1/8) = (16/8) – (1/8) = 15/8 = 30/16 = 45/24, and so on.

STRATEGY #2

ω| x ⊢ = (1/6) x (1/7) = 1/42. So 1 – [(1/6) x (1/7)] = (42/42) – (1/42) = 41/42.

6. Set the Goal as 41 ÷ 42 or 82 ÷ 84 (depending on the available cubes).
7. Write the Solution 1 – (ω| x ⊢).
8. Move any other ÷ signs to Forbidden as soon as possible.

OFFSHOOTS OF STRATEGY #2

9. 1 + (ω| x ⊢) = 1 + (1/42) = (42/42) + (1/42) = 43/42 = 86/84.
10. 2 – (ω| x ⊢) = 2 – (1/42) = (84/42) – (1/42) = 83/42.

EXERCISES

Note: For all these Exercises, assume the Sideways variation is in effect.

Give the value of each expression as a fraction.

1. 1 – ⊢ ________ **2.** 2 – ω| ________ **3.** 3 + ഗ ________

4. With all the strategies listed above, what cubes should you move to Forbidden as quickly as possible after you set the Goal? ____________

*Use Strategy **#1** or its offshoots to write a **three**-cube Solution for each Goal.*

5. 20 ÷ 25 ________ **6.** 72 ÷ 81 ________ **7.** 64 ÷ 56 ________

8. 65 ÷ 30 ________ **9.** 51 ÷ 27 ________ **10.** 91 ÷ 28 ________

*Use Strategy **#2** or its offshoots to write a **five**-cube Solution for each Goal.*

11. 78 ÷ 80 ________ **12.** 95 ÷ 48 ________ **13.** 72 ÷ 70 ________

14. 56 ÷ 54 ________ **15.** 70 ÷ 72 ________ **16.** 53 ÷ 18 ________

EQUATIONS WORKSHEET

7D

NAME ____

SIDEWAYS - IV

PRINCIPLE

Several more strategies with Sideways are offshoots of the ones in Worksheets **7B** and **7C**.

RECALL

⟨7⟩ + ⟨8⟩ = 15 ÷ 56 (15 = 7+8, 56 = 7x8)

⟨5⟩ + ⟨9⟩ = 14 ÷ 45 (14 = 5+9, 45 = 5x9)

⟨3⟩ − ⟨7⟩ = 4 ÷ 21 (4 = 7−3, 21 = 7x3)

OFFSHOOTS OF THE STRATEGY

1. After adding or subtracting the two unit fractions, as shown above, subtract the answer from 1.

   EXAMPLE 1

   1 − (⟨7⟩ + ⟨8⟩) = 1 − (15/56) = (56/56) − (15/56) = 41/56
   └ 56-15 (denominator − numerator)

   Reverse the steps above. Make 41 ÷ 56 the Goal with Solution of 1 − (⟨7⟩ + ⟨8⟩).

2. As in offshoot #1, add or subtract the unit fractions. But then *add* the answer to 1.

   EXAMPLE 2

   1 + (⟨5⟩ − ⟨9⟩) = 1 + (4/45) = (45/45) + (14/45) = 59/45
   └ 45+14 (denominator + numerator)

   Set 59 ÷ 45 as the Goal with Solution 1 + ⟨5⟩ + ⟨9⟩.

To figure out the Solution when the opponent sets a Goal like this, follow these steps.

a. Suppose the Goal is <u>58 ÷ 40</u>.
b. First, reduce the fraction to 29 ÷ 20.
c. Key on the denominator and pick two factors of 20: 4 and 5.
d. ⟨4⟩ + ⟨5⟩ = 9 ÷ 20.
e. Since the Goal equals 29 ÷ 20, the Solution must be 1 + ⟨4⟩ + ⟨5⟩.
Note: If the Goal equaled 11 ÷ 20, the Solution would be 1 − (⟨4⟩ + ⟨5⟩).

EXERCISES

Note: For all these Exercises, assume Sideways is in effect.

Give the value of each expression as a fraction.

1. 1 − (⟨7⟩+⟨5⟩) ________ **2.** 1 − (⟨2⟩+⟨9⟩) ________ **3.** 1 − (⟨4⟩−⟨7⟩) ________

4. 1 + (⟨8⟩+⟨3⟩) ________ **5.** 1 + (⟨2⟩−⟨5⟩) ________ **6.** 1 + (⟨3⟩−⟨8⟩) ________

*Write a **five**-cube Solution for each Goal.*

7. 35 ÷ 98 ________ **8.** 75 ÷ 54 ________ **9.** 82 ÷ 84 ________

10. 44 ÷ 84 ________ **11.** 69 ÷ 54 ________ **12.** 62 ÷ 60 ________

EQUATIONS WORKSHEET

7G

NAME ______________________________

UPSIDE DOWN – I

PRINCIPLE
The Upside-down variation states: "A cube representing a number may be used upside-down in the Goal or Solution to equal the additive inverse of that number.
NOTE: This variation is always in effect in Junior and Senior Divisions.

- The **additive inverse** of a number is its *opposite*. So the additive inverse of 2 is -2, the additive inverse of -7 is 7, and so on.
- You may *not* use an upside-down cube as part of a two-digit number. So expressions like 1ϛ, Ɩ5, and ϛƖ are not allowed.
- When you write an upside-down cube in an Equation, do it like this: $\underset{ud}{2}$

EXAMPLES

a. 7 + Ɛ = 7 + (-3) = 4
b. 7 – Ɛ = 7 – (-3) = 7 + 3 = 10
c. ㄥ + Ɛ = -7 + (-3) = -10
d. ㄥ – Ɛ = -7 – (-3) = -7 + 3 = -4
e. 4 x ϛ = 4 x (-5) = -20
f. ㄣ x ϛ = (-4) x (-5) = 20
g. $\underline{6}$ ÷ ᄅ = $\underline{6}$ ÷ (-2) = -3
h. $\overline{9}$ ÷ ᄅ = (-6) ÷ (-2) = 3

STRATEGY BASED ON THIS VARIATION

Use Upside-down to add without a + sign, as in Examples **b** and **d** above. If you have a – cube, play all + signs to Forbidden.

EXERCISES

Write the additive inverse (opposite) of each number.

1. 8 _____ **2.** -4 _____ **3.** 0 _____ **4.** 1 ÷ 4 _____

Write the value of each expression.

5. 7 + ᄅ _____ **6.** 7 – ᄅ _____ **7.** ㄥ + ᄅ _____ **8.** ㄥ – ᄅ _____

9. 3 x $\overline{6}$ _____ **10.** Ɛ x $\overline{6}$ _____ **11.** $\underline{9}$ ÷ Ɛ _____ **12.** $\overline{6}$ ÷ Ɛ _____

Give every possible value of each Goal.

13. 7–Ɛ+5 ______ **14.** 7–Ɛ–ᄅ ______ **15.** $\underline{6}$xㄣ÷2 ______ **16.** 8xㄣx2 ______

*With Upside-down, write a Solution for each Goal using the cubes listed in Required and **one** more cube. Assume the cube you need is available in Resources.*

	Goal	Required	Solution		Goal	Required	Solution
17.	1$\underline{6}$	$\underline{9}$, –	__________	**18.**	$\underline{6}$	8, +	__________
19.	ㄥ	1, +	__________	**20.**	$\overline{6}$	3, 3	__________
21.	ᄅ x $\underline{6}$	3, x	__________	**22.**	30 ÷ ϛ	+, 2	__________

***Elementary** only: Circle the number of each expression that is **not** allowed in Elementary Division.*

23. Ɛ ^ 0 **24.** Ɩ √ ᄅ **25.** ㄣ x ϛ **26.** 5 ^ ᄅ

EQUATIONS WORKSHEET

7J

NAME ____________________

0 WILD - I

PRINCIPLE

The 0 wild variation says: "The 0 cube may represent any numeral on the cubes, but it must represent the same numeral everywhere it occurs (Goal and Solution). Each Equation-writer must specify in writing the interpretation of the 0 cube if it stands for anything other than 0 in the Equation." NOTE: In Middle, Junior, and Senior Divisions, 0 may also represent an **operation**. Further, in Junior and Senior Divisions, players may choose either 0 or x as the wild cube.

EXAMPLES

a. If 0 wild is chosen, a Goal of 20 may represent 20, 21, 22, 23, 24, 25, 26, 27, 28, or 29. The Equation-writer indicates which value the Goal equals for his/her Solution.

b. (0 x 6) – 0 = 15 where both 0's stand for 3 is allowed.

c. (0 x 6) – 0 = 14, where the first 0 stands for 3 and the second 0 stands for 4, is *not* allowed.

d. (0 x 6) – 0 = 12, where the first 0 stands for 2 and the second 0 stands for 0, is *not* allowed.

e. Middle/Junior/Senior: In the Goal **506**, 0 *must* be interpreted as an operation sign since three-digit numerals are not allowed.

EXERCISES

Assume 0 wild is in effect.

Write in simplified form all possible values of each Goal.

1. 30 ____________ **2.** 03 ____________

3. 0 ÷ 2 ____________ **4.** 10+03 ____________

5. 1 ÷ 0 ____________ **6.** 0 x 0 ____________

Circle the number of each Goal in which 0 must be interpreted as a digit other than 0.

7. 7 ÷ 0 **8.** 0 ÷ 5 **9.** 0 √ 3 **10.** √(0 – 5)

11. 8 ^ 0 **12.** 3√(0–5) **13.** 07 **14.** 0 ÷ 0

The remaining exercises are for ***Middle/Junior/Senior*** *Divisions.*

Write in simplified form all possible values of each Goal.

15. 408 ____________

16. 20408 ____________

17. What variations would have to be in effect for the Goal **60003** to have a legal interpretation?

____

EQUATIONS WORKSHEET

NAME ____

7K

0 WILD - II

PRINCIPLES

1. There are various ways to indicate what 0 stands for in an Equation.
2. The 0 wild variation can be used in combination with the Sideways and Upside-down variations.

EXAMPLES

a. Here are some acceptable ways of indicating what 0 stands for in an Equation, with the two recommended methods shown first.

(7 x 3) + 4 = 25 (arrow from 0 below, pointing up to the 3)

(7 x 0) + 4 = 25 (arrow from 3 below, pointing up to the 0)

(7 x 3) + 4 = 25 (arrow from 0 above, pointing down to the 3)

(7 x 0) + 4 = 25 0 = 3

(7 x 0) + 4 = 25 (arrow from 3 above, pointing down to the 0)

(7 x 3) + 4 = 25 3 = 0

b. You can tell whether a 0 in the Goal is right-side up or sideways since each 0 on the cubes is taller than it is wide. Right-side up: **0** Sideways: ⬭

c. Suppose both 0 wild and Sideways are in force. Then, in the Goal or Solution, a player may use a 0 for, say 5, and turn in sideways to represent 1/5. Any other 0 in the Equation must represent 5. However, it may be used right-side up or sideways to give 5 or 1/5.

d. If 0 wild and Upside-down are both in force, a 0 may be used as, say, 4 and turned upside-down to obtain -4. Any other 0 in the Equation must be a 4 but may be used right-side up or upside-down.

e. If 0 wild and Upside-down are in force, a 0 in the Goal that is not part of a two-digit number may be interpreted as right-side up or upside-down.

f. If 0 wild, Sideways, and Upside-down are all in play, one 0 could be used for 1/3, another for -3, and another for 3.

EXERCISES

Assume both 0 wild and Sideways are in effect. Write all possible values of each Goal.

1. ⬭ + 7 ____________ **2.** 18 x ⬭ ____________

3. ⬭ √ 2 ____________ **4.** 16 ^ ⬭ ____________

5. 28 ÷ ⬭ ____________ **6.** 10 – ⬭ ____________

Assume both 0 wild and Upside-down are chosen. Write all possible values of each Goal.

7. 0 – 6 ____________ **8.** 12 ÷ 0 ____________

9. 2 √ 0 ____________ **10.** 0 ^ 3 ____________

11. 20 + 0 ____________ **12.** (0–0)÷0 ____________

EQUATIONS WORKSHEET

7L

NAME ____

0 WILD - III

PRINCIPLES

With 0 wild, certain Goals can be set that “sucker” an opponent into writing a Solution that uses 0 the wrong way.

EXAMPLE

Suppose the Goal is **(3 – 3) ÷ 0**, and a 0 is in Resources. An opponent may write a Solution like 0 ÷ 5 or 0 x 6 in which 0 equals 0. However, this is incorrect. The 0 in the Goal may *not* be 0 since it is in the denominator. All 0’s in the Goal and Solution must equal the same digit.

EXERCISES

Assume 0 wild is in effect.

1. Suppose the Goal is **8√(0–9)**. What can you conclude about 0’s for this shake?

____

2. Suppose the Goal is **(0–0)÷0**. There is another 0 in Resources. Can that 0 be used as 0 in a Solution? Explain.

____

3. Write all possible values of this Goal: **00÷0÷0** ____

____

MORE CHALLENGING EXERCISES

*Use **all** the Resources listed to write a Solution for each Goal. 0 is wild.*

	Goal	Resources	Solution
4.	45	+ x 0 0 4	________
5.	1÷4	÷ ^ 0 0 1	________
6.	42	– ^ 0 0 2	________
7.	6	– + √ 0 5 8 9	________
8.	13+0	– – + 0 2 2 5	________

*For Exercises **9-12**, assume **Sideways** is in force in addition to 0 wild.*

	Goal	Resources	Solution
9.	09÷40	+ – 0 3 7	________
10.	54 (MJS only)	x √ 0 3 6	________
11.	⬭ √2 (MJS)	– – x 0 2 2 4	________
12.	32^⬭ (MJS)	– + ÷ 0 0 3 6	________

*For Exercises **13-16**, assume **Upside-down** is in force in addition to 0 wild.*

	Goal	Resources	Solution
13.	Ɩ÷0	x ÷ ÷ 0 3 6 8	________
14.	0 + $\overline{6}$	+ ÷ 0 2 3	________
15.	7^ᘔ (MJS)	+ x ÷ 0 1 1 6	________
16.	0√ 3 (MJS)	÷ √ 3 4 9	________

EQUATIONS WORKSHEET

7M

NAME ____

MULTIPLE OPERATIONS

PRINCIPLE
The **multiple operations** variation says: "Any operation sign not in Forbidden (or the Goal) may be used many times in any Solution. If the Factorial variation is also chosen for the shake, an unlimited number of factorial operators may be used in each Solution. At most two factorials may be used in the Goal."

COMMENTS

a. An Equation-writer does not have to indicate in any special way that an operation cube is used two or more times. Simply write the symbol wherever you want it in the Solution.

b. After a Now challenge, one operation cube in Resources may be used multiple times in the Solution since Resources cubes may be treated like Permitted cubes after challenges.

c. After an Impossible challenge, all cubes still in Resources are equivalent to Permitted cubes. Therefore, any operation signs in Resources when an Impossible challenge is made may be used many times in a Solution.

d. Middle/Junior/Senior: If 0 (or x in Jr/Sr) is wild, each 0 (or x) cube may be used multiple times in any Solution only if it represents an operation.

e. Senior: If x (or 0) wild and √ = *i* are also chosen, x (or 0) used as *i* may *not* be used multiple times since *i* is a numeral, not an operation.

EXERCISES

*Circle the number of each true statement. Assume **multiple operations** is chosen.*

1. Any operation cube in Required or Permitted may be used many times in any Solution.
2. After a Now challenge, a + in Resources may be used multiple times in any Solution.
3. If a player challenges Impossible, any ^ in Resources may be used many times in Solutions.
4. An Equation-writer must indicate in a special way that an operation cube is used many times.
5. If Factorial is also in effect, players are still limited to two factorials in their Solutions.
6. MJS: If 0 wild has also been chosen, 0 may be used multiple times as an operation sign in any Solution.
7. JS: With x wild, x may be used many times in Solutions regardless of what it represents.
8. Senior: If 0 wild and √ = *i* are chosen, 0 may be used multiple times as *i*.

MORE CHALLENGING EXERCISES (MJS ONLY)

*Use **all** the Resources listed to write a Solution for each Goal. Assume multiple operations and sideways are in effect.*

	Goal	Resources	Solution
9.	16 ^ ω	^ ÷ 3 4 8	________________
10.	2^(ω+⊣)	÷ √ 2 2 2 3 7	________________

EQUATIONS WORKSHEET

7T

NAME ____

NUMBER OF FACTORS - I

PRINCIPLE

The **Number of Factors** variation says: "x*A* means the number of counting number factors of *A*, where A is a counting number. In Elementary Division, *A* must not be bigger than 200."

The "counting numbers" are 1, 2, 3, 4, 5, ... (*not* 0).

EXAMPLES

a. x(6x2) = x12 = 6 since the factors of 12 are 1, 2, 3, 4, 6, and 12.
└ because of its placement, this x means multiplication. It has numbers in front of it and behind it.

b. x(4x4) = x16 = 5 because the factors of 16 are 1, 2, 4, 8, and 16.

c. These expressions have no value: x0, x(1÷2), and x(1–4).
In **Elementary** Division, x(5^4) has no value since 5^4 is bigger than 200.

d. xx9 = x(x9) = x3 (since 9 has 3 factors) = 2 (since 3 has 2 factors).

e. A Goal of **x4x8** has two interpretations. In both interpretations, the first x means number of factors, and the second x means multiplication.

- (x4)x8 = 3 x 8 [since 4 has 3 factors] = 24. Note: (x4)x8 is the *default* interpretation if the Equation-writer does not use parentheses.
- x(4x8) = x32 = 6 [1, 2, 4, 8, 16, and 32 are the factors of 32.]

EXERCISES

Write all possible values of each Goal. Assume Number of Factors has been chosen.

1. x1 ______	**2.** x2 ______	**3.** x4 ______	**4.** x6 ______
5. x7 ______	**6.** x9 ______	**7.** x13 ______	**8.** x16 ______
9. x21 ______	**10.** x24 ______	**11.** x29 ______	**12.** x36 ______
13. xx6 ______	**14.** xx25 ______	**15.** xx31 ______	**16.** xxx64 ______
17. x5x2 ______	**18.** x6x2 ______	**19.** x8+7 ______	**20.** x5–7 ______

Examples x5÷6 = (x5)÷6 = 2÷6 = 1/3   Note: x(5÷6) has no meaning.
x3^3 = (x3)^3 = 2^3 = 8 or x3^3 = x(3^3) = x27 = 4 [1, 3, 9, 27]

21. x4÷8 ______	**22.** x2^4 ______	**23.** 4^x5 ______	**24.** x4√8 ______
25. √x8 ______	**26.** x7+x9 ______	**27.** xx9+x8 ______	**28.** x5-xx6 ______

29. Which group of counting numbers has an *odd* number of factors? ________________

MORE CHALLENGING EXERCISES

Write all possible values of each Goal in your division.

30. x2^7 ________________   **31.** x9x11 ________________

32. x8^25 ________________   **33.** x42x8 ________________

EQUATIONS WORKSHEET

7U

NAME ______________________________

NUMBER OF FACTORS – II

PRINCIPLE
To compute the number of factors of larger numbers, use a systematic method.

EXAMPLES

a. Suppose the Goal is **x 2^5**. Then the factors of 2^5 are: 2^0, 2^1, 2^2, 2^3, 2^4, and 2^5. So 2 to the *fifth* power has *six* factors (because 2^0, which is 1, is a factor).

b. Suppose the Goal is x72 . Here is a systematic way to find the number of factors of 72.

- Factor 72 into its prime factors. $72 = 8 \times 9 = 2^3 \times 3^2$
- Based on Example **a**, 2^3 has *4* factors, and 3^2 has *3* factors. So 72 has 4 x 3 or **12** factors. To verify this answer, list the 12 factors like this.

$2^0 \times 3^0 = 1$	$2^1 \times 3^0 = 2$	$2^2 \times 3^0 = 4$	$2^3 \times 3^0 = 8$
$2^0 \times 3^1 = 3$	$2^1 \times 3^1 = 6$	$2^2 \times 3^1 = 12$	$2^3 \times 3^1 = 24$
$2^0 \times 3^2 = 9$	$2^1 \times 3^2 = 18$	$2^2 \times 3^2 = 36$	$2^3 \times 3^2 = 72$

c. Suppose the Goal is **x72^4**. This Goal has two interpretations in Middle Division.

- (x72)^4 = 12^4 [from Example **b** above].
- **MJS** only: x(72^4). To determine the number of factors of 72^4, use the prime factors from Example **b** like this.

$72^4 = (2^3 \times 3^2)^4 = (2^3)^4 \times (3^2)^4 = 2^{12} \times 3^8$
So x(72^4) = 13 x 9 = **117**. ← add one to each exponent and multiply

EXERCISES

With number of factors, write all values of each Goal for your division.

1. x36 _____ **2.** x88 _____ **3.** x96 _____ **4.** x80 _____

SAMPLE Goal: xx48 ("# factors of the # factors of 48")
Values xx48 = x(x48) = x[x(2^4x3^1)] = x(5x2) = x10 = 4

5. xx56 _____ **6.** xx78 _____ **7.** xx42 _____ **8.** xx90 _____

SAMPLE Goal: x9^24
Values (x9)^24 = 3^{24}; **MJS**: x(9^24) = x[$(3^2)^{24}$] = x(3^{48}) = 49

9. x2^13 _____ **10.** x3^11 _____ **11.** x8^9 _____ **12.** x6^12 _____

SAMPLE Goal: x12x16
Values (x12)x16 = x(2^2x3^1)x16 = 6 x 16 = **96**; **MJS**: x(12x16) = x[2^2x3^1x2^4] = x(2^6x3^1) = **14**

13. x45x9 _____ **14.** x7x30 _____ **15.** x45x9 _____ **16.** x12x9 _____

*The remaining Exercises are **Middle/Junior/Senior** only.*

SAMPLE Goal: √[x(7^48)]
Value √[x(7^48)] = √49 = **7** In general, √[x(*p*^(*s*-1))] = √*s* where *p* is prime, *s* is a square.

17. √[x(3^63)] _____ **18.** √[x(37^8)] _____ **19.** √[x(53^3)] _____ **20.** √[x(5^99)] _____

EQUATIONS WORKSHEET

NAME ______________________________

7AC

FACTORIAL – V

PRINCIPAL
A **Factorial** can be used in Goals involving roots.

EXAMPLES – Assume Factorial has been chosen.

a. Suppose the Goal is $\sqrt{3\times24}$. $\sqrt{3!\times24} = \sqrt{3\times2\times3\times8} = \sqrt{9\times16} = 3 \times 4 = 12$.
What other numbers could have been multiplied times 3! to give a perfect square in the Goal of Example **a**? $\sqrt{3!\times\ ?\ } = \sqrt{(3\times2\times?)}$ ← Replace the ? with a number that produces a perfect square when multiplied by 6.
Multiply by 6 to get $\sqrt{(3\times2\times6)} = \sqrt{36} = 6$. Also, $\sqrt{(3!\times54)} = \sqrt{(3\times2\times6\times9)} = \sqrt{(36\times9)} = 6 \times 3 = 18$.

b. Suppose the Goal is of the form $\sqrt[3]{(4!\times\_\_)}$. A number must fill the blank to create a *perfect cube*.
What numbers will work in the blank?
$\sqrt[3]{(4!\times\_\_)} = \sqrt{4\times3\times2\times?}$ 4! contains an 8, which is the cube of 2. That leaves the 3 needing a "partner" to produce a perfect cube. The smallest "partner" is 9 because $9 \times 3 = 27$.
$\sqrt[3]{(4!\times9)} = \sqrt[3]{(4\times3\times2\times9)} = \sqrt[3]{(4\times2\times3\times9)} = \sqrt[3]{(8\times27)} = 2 \times 3 = 6$

EXERCISES

1. What number other than 6, 24, and 54 can be placed in the blank in $\sqrt{(3!\times\_\_)}$ to produce a whole number? Show the work to produce the value for that number.

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2. What other number besides 9 can be placed in the blank in $\sqrt[3]{(4!\times\_\_)}$ to produce a whole number result? Show the work to produce the value for that number.

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Assume Factorial is in effect. Give all whole numbers that can fill the blank to produce a whole number value when a factorial is placed behind the first number after the root sign. In each case, give the number placed in the blank and the value that results.

3. $\sqrt{4!\times\_\_}$ ______________________________

4. $\sqrt{6!\times\_\_}$ ______________________________

5. $\sqrt{5!\times\_\_}$ ______________ **6.** $\sqrt{7!\times\_\_}$ ______________

7. $\sqrt{8!\times\_\_}$ ______________________________

8. $\sqrt{9!\times\_\_}$ ______________ **9.** $\sqrt[3]{3!\times\_\_}$ ______________

10. $\sqrt[3]{4!\times\_\_}$ ______________ **11.** $\sqrt[4]{4!\times\_\_}$ ______________